4.2.3 Properties of matter waves

The following are the properties associated with the matter waves

1. Matter waves are associated only with particles in motion
2. They are not electromagnetic in nature
3. Group velocity is associated with matter waves
4. As a result of superposition of large number of componenet waves which slightely differ in frequency, matter waves are localized.
5. The phase velocity has no physical meaning for matter waves
6. The amplitude of the matter wave at a given point is associated with the probability density of finding the particle at that point.
7. The wave length of matter waves is given by $\lambda = \frac{h}{mv}$

4.3 Heisenberg's Uncertainty Principle

Statement: The simultaneous determination of the exact position and momentum of a moving particle is impossible.

Explanation : According to this principle if Δx is the error involved in the measurement of position and Δp_x is the error involved in the measurement of momentum during their simultaneous measurement, then the product of the corresponding uncertainties is given by

$$\Delta x \Delta p_x \geq \frac{h}{4\pi} \tag{4.3}$$

$$\Delta E \Delta t \geq \frac{h}{4\pi} \tag{4.4}$$

$$\Delta \theta \Delta L \geq \frac{h}{4\pi} \tag{4.5}$$

The product of the errors is of the order of Planck's constant. If one quantity is measured with high accuracy then the simultaneous measurement of the other quantity becomes less accurate.

Physical significance : According to Newtonian physics the simultaneous measurement of position and momentum are ***exact***. But the existence of matter waves induces serious problems due to the limit to accuracy associated with the simultaneous measurement. Hence the ***exactness*** in Newtonian physics is replaced by ***probability*** in quantum mechanics.

4.3.1 Application of uncertainty principle

Non-exixtance electrons inside the nucleus : Beta rays are emitted by the nucleus. When it was first observed it was believed that electrons exist inside the nucleus and are emitted at certain instant. If the electron can exist inside the atomic nucleus then uncertainty in its position must not exceed the diameter of the nucleus. The diameter of the nucleus is of the order of Δx_{max} is $10^{-14}m$. Applying Heisenberg's uncertainty principle for an electron expected to be inside the nucleus we get

$$\Delta x_{max} \Delta p_{min} \geq \frac{h}{4\pi} \tag{4.6}$$

$$\Delta p_{min} \geq \frac{h}{4\pi \Delta x_{max}} \tag{4.7}$$

$$\Delta p_{min} \geq \frac{6.625 \times 10^{-34}}{4 \times 3.142 \times 10^{-14}} = 5.276 \times 10^{-21} kgms^{-1} \tag{4.8}$$

Therefore, the electron should possess momentum

$$p_{min} \simeq \Delta p_{min} = 5.276 \times 10^{-21} kgms^{-1} \tag{4.9}$$

Relativistically the energy of the electron is given by the equation

$$E = \sqrt{p^2c^2 + m_0{}^2c^4} \tag{4.10}$$

Since, $m_0{}^2c^4 \ll p^2c^2$, we get $E = pc$. Therefore

$$E_{min} = p_{min}c = 5.276 \times 10^{-21} \times 3 \times 10^8 \tag{4.11}$$

$$E_{min} = 1.58 \times 10^{-12} J \tag{4.12}$$

$$E_{min} = \frac{1.58 \times 10^{-12}}{1.6 \times 10^{-19}} = 9.9\textbf{MeV} \tag{4.13}$$

Conclusion : *According to experiments, the energy associated with the beta ray (electron) emission is around 3 MeV which is much lesser than the energy of the electron expected to be inside the nucleus 9.9 MeV. Hence electrons do not exist inside the nucleus.*

Note : Equations 4.4 and 4.5 represent the uncertainty relationship between the conjugate physical quantities (Energy,time) and (Angualr displacement, Angular momentum).

4.4 Wave Function

According to the de Broglie's hypothesis the relation between momentum and wavelength is found to be experimentally valid for both photons and particles. The quanta of matter or radiation can be represented in agreement with uncertainty principle by wave packets. Thus it suggests

Part III

Module - 3 - LASER & Optical Fibers

Chapter 5

LASER and its Applictions

5.1 Introduction

LASER is an acronym for Light Amplification by Stimulated Emission of Radiation. LASER is a highly monochromatic, highly coherent, highly directional and high intensity beam of light. The first LASER was built by Theodore H Maiman in the year 1960. Thus it finds various applications starting from industries to communication.

5.2 Interaction of radiation with matter

The interaction between radiation and matter occurs through the following three processes.

1. Induced absorption
2. Spontaneous emission
3. Stimulated emission

5.2.1 Induced Absorption

When a photon of right energy is incident on the atom then the photon is absorbed This process is induced by the photon and hence it is called Induced Absorption.

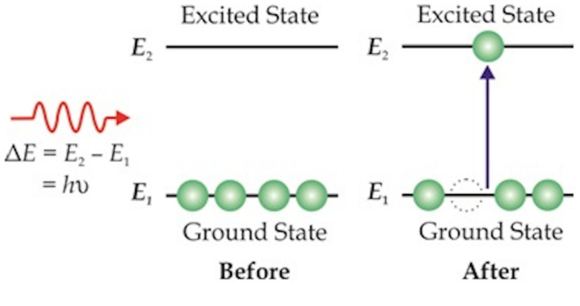


Figure 5.1: Induced absorption

Consider an atom in a lower energy states E_1, it will excite to higher energy states E_2 by absorbing the incident photon of energy $E = h\nu = E_2 - E_1$. Here E_1 energy of the lower energy state, E_2 is the energy of the higher energy state, h is the Planck's constant ν is the frequency of photon.

5.2.2 Spontaneous Emission

Spontaneous emission is the process of emission of photon, when an atom transits from higher energy level to lower energy level without the influence of any external energy.

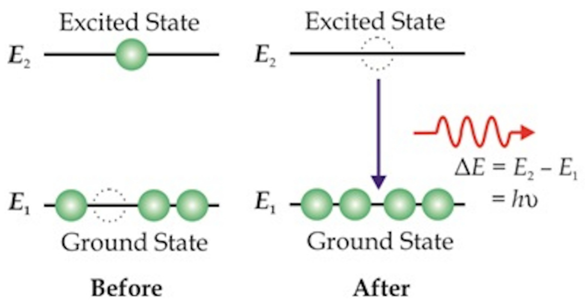


Figure 5.2: Spontaneous emission

An electron in the higher energy state of an atom makes a transition to lower energy state without the action of any external agency. the energy of the photon emitted is given by $E = h\nu = E_2 - E_1$. In this process the emitted photons need not travel in the same direction. Thus the light beam is not directional.

5.2.3 Stimulated Emission

When a photon of suitable energy interacts with an atom in the higher energy state then the atom is stimulated (Forced) to make transition from higher energy state to a lower energy state with the emission of a photon. Both the incident photon and the emitted photons are coherent and travel in the same direction. Thus the process is called stimulated emission.

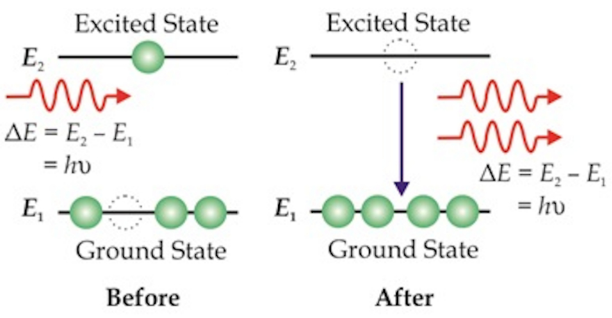


Figure 5.3: Stimulated emission

When a photon of energy $h\nu = E_2 - E_1$ interacts with an atom in the higher energy state the stimulated emission takes place with the emission of two photons of same energy that are highly directional and coherent.Thus stimulated emission could be used to generate a highly coherent directional beam of light.

5.3 LASER Action and the Conditions for LASER action

Consider a LASER system. Let an atom in the excited state is stimulated by a photon of right energy so that atom makes stimulated emission. Two coherent photons are obtained. These two coherent photons if stimulate two atoms in the exited state to make emission then four coherent photons are emitted. These four coherent photons stimulate 4 more atoms in the excited state resulting in 8 coherent photons and so on. As the process continues number of coherent photons increases. These coherent photons constitute an intense beam of LASER. This phenomenon of building up of number of coherent photons so as to get an intense LASER beam is called lasing action.

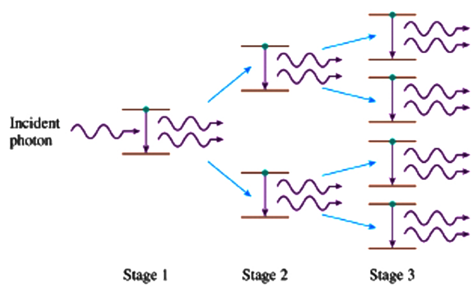


Figure 5.4: LASER action

LASER action could be achieved through the conditions population inversion and metastable state.

5.3.1 Population inversion & Meta-stable state

Population Inversion

If a system is under thermal equilibrium the number of atoms in excited state is less than the number of atoms in the lower energy state. For the production of LASER number of stimulated emission must be more when compared to induced absorption and spontaneous emission. This is possible only if the number of atoms in the higher energy state is more than the number of atoms in the lower energy state and is called population inversion. The means of achieving population inversion by supplying energy from a suitable source is called Pumping. In addition,to have more stimulated emissions, the life time of atoms in the excited state must be longer. Thus the essential conditions for population inversion are

1. Higher energy state should posses a longer life time.
2. The number of atoms in the higher energy state must be greater than the number of atoms in the lower energy state.

Metastable state

The life time of an energy level is of the order of 10^{-8} second. If an atom posses unusual longer life time in an energy state such a state is referred to as a metastable state. Usually the life time of metastable state varies from 10^{-2}s to 10^{-3}s. Population inversion could be achieved with the help of three energy state with one of them a metastable state and is as shown in the figure 5.5. The population inversion is achieved between the state E_2 and E_1 as state E_2 is a metastable state.

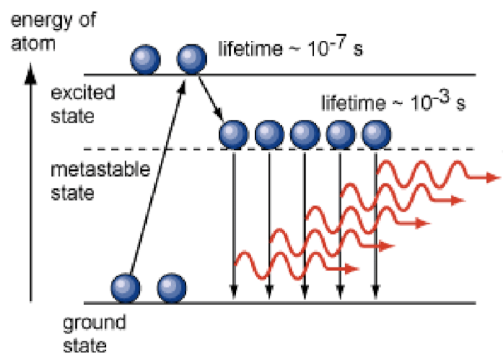


Figure 5.5: Population inversion, E_2 is Metastable state

Note : The principles of Laser are

1. Stimulated Emission
2. Population Inversion
3. Metastable State

5.3.2 Requisites of a LASER system

The three requisites of a LASER system are,

1. Excitation source for pumping action
2. Active medium that supports metastable states
3. LASER cavity

Pumping

In order to achieve population inversion more and more atoms are to be moved to higher energy state and is called pumping. This is achieved by supplying suitable energy using an energy source. If optical energy is used then the pumping is called optical pumping and if electrical energy is used then the pumping is called electrical pumping.

Active medium

Population inversion occurs at certain stage in the active medium due to the absorption of energy. The active medium supports metastable states. After this stage the active medium is capable of emitting LASER light.

Resonant cavity (or) LASER cavity

The LASER Cavity consists of an active medium bound between two highly parallel mirrors. The reflection of photons from the mirrors makes the multiple traverse of photons through the active medium inducing more and more stimulated emissions. Thus amplification of light is achieved. This also helps to tap certain permissible part of LASER energy from the active medium.The cavity resonates and the output will be maximum when the distance L between the mirrors is equal to an integral multiple of $\frac{\lambda}{2}$. Here λ is the wavelength of incident suitable radiation.the length of the LASER cavity is expressed as

$$L = \frac{n\,\lambda}{2} \tag{5.1}$$

5.4 Einstein's A and B co-efficients and expression for energy density

Consider a system containing N atoms and is under thermal equilibrium. Let E_1 and E_2 be the lower and higher energy levels that contain N_1 and N_2 number of atoms respectively. Let the incident energy density of the radiation be E_ν. Hence the system absorbs and emits the energy through the following processes. The energy of the photons absorbed and emitted by the atoms is $E = h\nu = (E_2 - E_1)$

Rate of induced absorption

The rate of induced absorption is defined as the number of induced absorption per second per unit volume in unit time. Rate of absorption depends on

1. Number of atoms in the lower energy state N_1.
2. The incident energy density E_ν.

Hence

1. Rate of Induced absorption $\propto N_1\, E_\nu$
2. Rate of Induced absorption = $B_{12}\, N_1\, E_\nu$

Here B_{12} is proportionality constant called Einsteins coefficient of Induced absorption.

Rate of spontaneous emission:

The number of spontaneous emission per unit volume in unit time is called rate of spontaneous emission. Rate of spontaneous emission depends on

Since spontaneous emission is a voluntary process it is independent of energy density E_ν. The rate of spontaneous emission depends only on the number of atoms in the higher energy state N_2. Thus

1. Rate of spontaneous emission $\propto N_2$
2. Rate of Spontaneous emission = $A_{21}\, N_2$

Here A_{21} is the proportionality constant called Einstein's co-efficient of spontaneous emission.

Rate of stimulated emission

The number of stimulated emission per unit volume in unit time is called rate of stimulated emission. Rate of stimulated emission depends upon,

1. Number of atoms in the higher energy state (N_2)
2. The energy density (E_ν).

Hence

1. The Rate of stimulated emission $\propto N_2\, E_\nu$
2. Rate of stimulated emission = $B_{21}\, N_2\, E_\nu$

Here the proportionality constant called B_{21} is Einstein's coefficient of stimulated emission.

Under Thermal Equilibrium the total Energy of the System remains unchanged. Hence Rate of Absorption is equal to rate of emission.

$\therefore$ Rate of Induced Absorption = [Rate of Spontaneous emission + Rate of Stimulated Emission]

$\therefore$

$$B_{12}\, N_1\, E_\nu = A_{21}\, N_2 + B_{21}\, N_2\, E_\nu \tag{5.2}$$

$$(B_{12}\,N_1 - B_{21}\,N_2\,)\,E_\nu = A_{21}\,N_2$$

$$E_\nu = \frac{A_{21}\,N_2}{B_{12}\,N_1 - B_{21}\,N_2} \quad (5.3)$$

$$E_\nu = \frac{A_{21}}{B_{12}\,\frac{N_1}{N_2} - B_{21}} \quad (5.4)$$

$$E_\nu = \frac{A_{21}}{B_{21}}\left[\frac{1}{\frac{B_{12}}{B_{21}}\,\frac{N_1}{N_2} - 1}\right] \quad (5.5)$$

According to Boltzmann relation the we have

$$\frac{N_2}{N_1} = e^{\frac{-h\nu}{kT}} \quad (5.6)$$

or we can re-write as,

$$\frac{N_1}{N_2} = e^{\frac{h\nu}{kT}} \quad (5.7)$$

Here h is the Planck's constant, c is the speed of light in vacuum, λ is the wavelength of the photon, k is the Boltzmann constant and T is the absolute temperature. Substituting for $\frac{N_1}{N_2}$ in equation 5.5

$$E_\nu = \frac{A_{21}}{B_{21}}\left[\frac{1}{\frac{B_{12}}{B_{21}}\left(e^{\frac{h\nu}{kT}}\right) - 1}\right] \quad (5.8)$$

According to Planck's radiation law, the equation for energy density in the frequency domain is given by

$$E_\nu = \frac{8\pi h\nu^3}{c^3}\left[\frac{1}{e^{\frac{h\nu}{kT}} - 1}\right] \quad (5.9)$$

on comparing equations 5.8 and 5.9 we can get

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \quad (5.10)$$

and

$$\frac{B_{12}}{B_{21}} = 1 \quad (5.11)$$

or $B_{12} = B_{21}$

This means that Probability of Induced absorption is equal to Probability of Stimulated emission. Hence A_{21} & B_{21} can be replaced by A & B. Thus equation 5.8 could be written as

$$E_\nu = \frac{A}{B}\left[\frac{1}{e^{\frac{h\nu}{kT}} - 1}\right] \quad (5.12)$$

Hence the expression for energy density in terms of Einstein's co-efficient A and B.

5.5 Construction and Working of CO_2 LASER

It is a molecular gas LASER and was designed by Dr. C.K.N. Patel of BELL labs in 1963. In molecular LASER, the lasing action occurs between vibration-rotation energy levels of the molecules. It operates in the middle IR region. it is a four level LASER producing both continuous and pulsed output wave-forms. It has very high efficiency of upto 30%. it is widely used in industry and medical applications.

CO_2 molecule has one carbon atom about which two oxygen atoms are symmetrically located. CO_2 molecule can vibrate in 3 different modes. In each mode, the centre of gravity remains same.

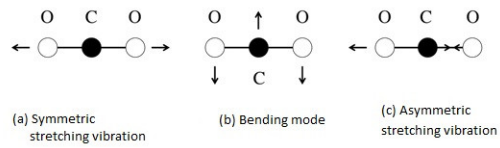


Figure 5.6: Mode of vibration

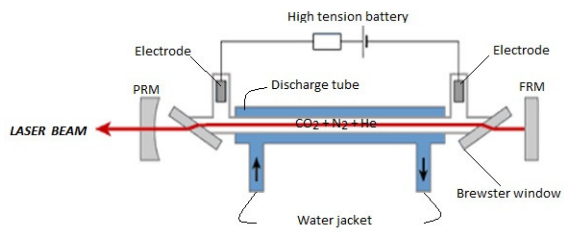


Figure 5.7: Carbon Dioxide LASER

Construction: A typical CO_2 gas LASER is as shown in figure. It consists of a long narrow discharge tube of diameter 2.5 cm and length about 5m made of fused quartz. The tube is maintained cool & is filled with active medium, a mixture of CO_2, N_2 & He gas in the ratio 1:2:3. Sometimes, water vapor is added to reoxidise. Depending on the usage either AC or DC voltage is applied between the electrode to provide Electrical discharge pumping. Brewster windows polarize the LASER beam.

Working:

When a high DC voltage is supplied across th electrodes, discharge of gases takes place. During the discharge electrons collide with N_2 and CO_2 molecules. Thus

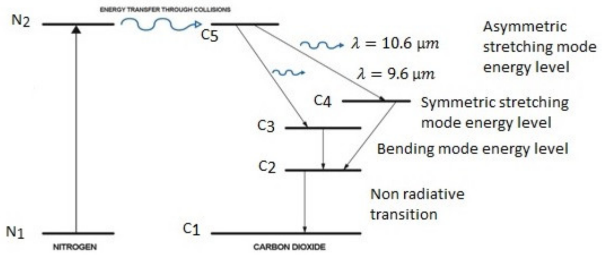


Figure 5.8: CO_2 LASER energy level diagram

the molecules absorb energy and are excited to higher energy level. This is called *Collision of First Kind.*

$$e^1 + N_2 \rightarrow N_2^* + e^2 \quad (5.13)$$

$$e^3 + CO_2 \rightarrow CO_2^* + e^4 \quad (5.14)$$

Since an higher energy level N_2 of Nitrogen molecule matches with one of the *vibration – rotation* level C_5 of CO_2 the resonance transfer of energy takes place. This is referred to as *Collision of Second Kind.*

$$N_2^* + CO_2 \rightarrow N_2 + CO_2^* \quad (5.15)$$

Due to continuous discharge population inversion is achieved between the level C_5 and C_4 also between the levels C_5 & C_3 as shown in the energy level diagram. The transition from C_5 to C_4 and C_3 results in the emission of LASER of wavelengths $10.6\mu m$ and $9.6\mu m$ respectively in the IR region of the EM spectrum. The system gets heated due to non-radiative transitions during the LASER action.

Advantages

1. It generates both continuous and pulsed LASER output.

2. It has High efficiency compared to other LASER devices

Applications:

1. It is used in industrial applications like welding, cutting, drilling , etc.

2. It is used in LIDAR due to minimum atmospheric attenuation.

3. It finds application in communication systems.

5.6 Semiconductor LASER or Diode LASER

Introduction

Semiconductor diode LASER is an LED with heavily doped P and N sections. First semiconductor LASER was fabricated in 1962 using $Ga - As$ by *Hall* with his co-workers. It is a low cost and high efficiency LASER.

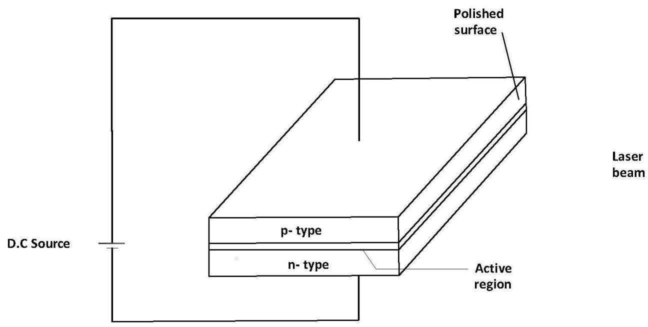
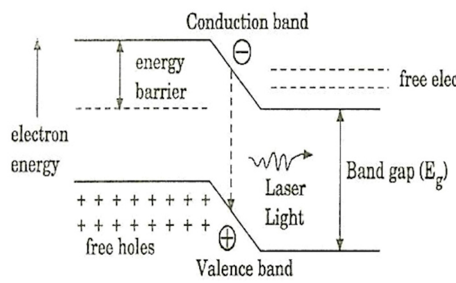


Figure 5.9: Semiconductor Diode LASER

Construction

Construction

The $Ga - As$ LASER diode belongs to direct band gap semiconductors. The *n*-section is derived by doping the substrate with *Tellurium* and *p*-section is derived by doping the substrate with *Zinc.* The diode used is in the form of a cube with dimension 0.4 mm. The depletion region is of thickness 0.1 micrometer and lies horizontal as shown in the figure 5.9.The current is passed through the ohmic contacts provided to the top and bottom faces. The front and back faces are polished and made highly parallel to each other to have a LASER cavity. The other two faces are roughened.



Working

The Diode is forward biased using an external source. Therefore electrons and holes flow across junction. Hence the current flows through the diode. When a hole meets an electron it recombines with electron emitting a photon. This could be considered as the transition of electron from conduction band to valance band. When the current is low spontaneous emission is predominant. If the current is sufficiently high population inversion is achieved. The photons liberated initially due to spontaneous emissions induce further stimulated emissions. The LASER cavity helps in the amplification of light. Finally this results in an avalanche of photons and hence the LASER action is achieved. If the GaAs semiconductor is used then the wavelength of the LASER emitted is 840nm.

Advantages

1. It has excellent efficiency
2. The output can be modulated
3. It produces both continuous wave output or pulsed output.
4. It is highly economical

Applications

1. It is used in optical fiber communication.
2. It is used in commercial CD recording and reading.

5.7 Applications of LASER

LASER has wide range of applications pertaining all disciplines of engineering. Here in the syllabus only two applications are discussed pertaining to defense and data storage.

5.7.1 Application of LASER in defense - LASER Range Finder

Ranging is finding the accurate distance of location of an object from a reference place. During the war the ranging has to be done with a very high accuracy so that shell hits and destroys the enemy tank successfully. The laser beam is ideally suited due to its high intensity and high directionality, It can travel a few kilometer with very low losses in intensity and very low divergence. The LASER range finders using neodymium and carbon dioxide LASERs have become a standard item for artillery and tanks.

A schematic diagram of a typical LASER range finder is as shown in the figure 5.10. The LASER range finder works on the principle of a radar and is called LIDAR which stands for Light Detection and Ranging. It makes use of the characteristic properties of the LASER beam, namely, monochromaticity, high intensity, coherency, and directionality. A collimated pulse of the LASER beam is

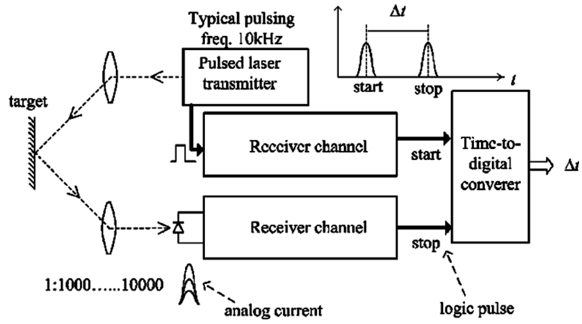


Figure 5.10: LASER Range Finder

directed towards a target and the reflected 1ight from the target is received by an optical system and detected. The time taken by the LASER beam for the to and fro travel from the transmitter to the target is measured. When half of the time thus recorded is multiplied by the velocity of light, the product gives the range.

5.7.2 Advantages

The advantages of LASER range finders over conventional range finders.

1. They are light weight.
2. They posses high reliability.
3. They possess superior range.

5.7.3 Application of LASER in Medicine - Eye and Skin Treatment

Eye Treatment: LASIK (Laser-assisted in situ keratomileusis) is commonly known as laser eye surgery or laser vision correction. It is a type of refractive surgery for the correction of myopia, hyperopia, and astigmatism. During LASIK surgery, a special type of cutting laser is used to precisely change the shape of the dome-shaped clear tissue at the front of your eye (cornea) to improve vision. An excimer laser (193 nm) remove a small amount of human tissue by vaporization.

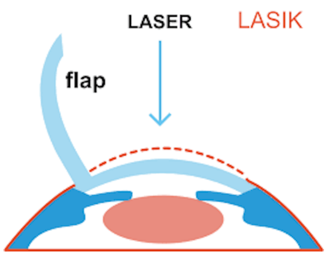


Figure 5.11: Graded index multimode fiber

Skin treatment: LASER is used in resurfacing treatment of various skin conditions including scars, vascular, pigmented lesions, texture, uneven tone, tattoo, wrinkle and hair removals. The treatment involves the application of a precision LASER beam to the skin. The LASER removes the superficial layers of skin and stimulates the production of a newer and healthier layer which leaves patients looking visibly younger. Care is taken such that there is no risk of the laser penetrating beyond a certain depth in skin tissue. Only the outermost, sun-damaged layers of skin are removed, leaving the deeper skin (dermis) to produce a new of skin above with improved elasticity,tone and texture.

5.7.4 Model Questions

1. What is LASER? Discuss the possible ways through which radiation and matter interaction can take place.
2. Explain the terms, (i) Induced absorption, (ii) Spontaneous emission, (iii) Stimulated emission, (iv) Population inversion, (v) Metastable state & (vi) Resonant cavity.
3. Explain requisites of LASER system.
4. Discuss the condition for Lasing action.
5. Obtain an expression for energy density of radiation in terms of Einstein's Co-efficients.
6. Describe the construction of CO_2 LASER & Explain its working with the help of energy level diagram. Discuss the different vibrational modes.
7. What is Semiconductor LASER? Describe with energy band diagram the construction & working of Semiconductor diode LASER along with applications.
8. Explain how data storage is achieved in a compact dics
9. Describe how a LASER range finder is made use of in defense.
10. Mention applications of LASER.

5.7.5 Numerical Problems

1. Find the ratio of population of two energy levels in a LASER if the transition between them produces light of wavelength 6493 Å, assuming the ambient temperature at 27°C.
2. Find the ratio of population of two energy levels in a medium at thermal equilibrium, if the wavelength of light emitted at 291 K is 6928 Å.
3. The ratio of population of two energy levels out of which one corresponds to metastable state is 1.059×10^{-30}. Find the wavelength of light emitted at 330 K.
4. Find the ratio of population of two energy levels in a medium at thermal equilibrium, if the wavelength of light emitted at 300 K is $10\mu m$. Also find the effective temperature when energy levels are equally populated.
5. The average power output of a LASER beam of wavelength 6500 Å is 10 mW. Find the number of photons emitted per second by the LASER source.
6. The average power of a LASER beam of wavelength 6328 Å is 5 mW. Find the number of photons emitted per second by the LASER source.
7. A pulsed LASER has an average power output 1.5 mW per pulse and pulse duration is 20 ns. The number of photons emitted per pulse is estimated to be 1.047×10^8. Find the wavelength of the emitted LASER.
8. A pulsed LASER with power 1 mW lasts for 10 ns. If the number of photons emitted per pulse is 5×10^7. Calculate the wavelength of LASER.
9. A Ruby LASER emits a pulse of 20 ns duration with average power per pulse being 100 kW. If the number of photons in each pulse is 6.981×10^{15}, calculate the wavelength of photons.
10. In a LASER system when the energy difference between two energy levels is 2×10^{-19} J, the average power output of LASER beam is found to be 4 mW. Calculate number of photons emitted per second.
11. Find number of modes of standing waves and their frequency separation in resonant cavity of length 1 m, in He-Ne LASER operating at wavelength of 6328 Å.
12. Calculate the ratio of i) Einstein coefficient $\left(\frac{A_{21}}{B_{12}}\right)$. ii) stimulated to spontaneous emission for wavelength $1.39\mu m$ at temperature 300 K.
13. A semiconductor LASER has a peak emission radiation of wavelength of 1.24 μm. What is its band gap value in eV?

Chapter 6

Optical Fibers

6.1 Introduction

Optical fibers are the wires and strands made of transparent dielectrics which guide light over longer distances using the phenomenon of ***Total Internal Reflection***. Many optical fibers are bundled together and are given a protective layer of covering using an insulating material. This bundle is called Optical Fiber Cable or Fiber Bundle (Bundle Fiber).

Construction: The sectional view of a typical optical fiber is as shown in the figure. It has three regions named Core, Cladding and Sheath.

1. The innermost light guiding region is called Core.

2. The layer covering core and helps in total internal reflection of light is called Cladding or Clad.

3. The outermost protective layer is called Sheath (Coating). The sheath protects the fiber from mechanical stress and chemical reactions.

The optical fiber is designed to support total internal reflection and hence the RI of coren_1 is made greater than the RI of cladding n_2. A typical fiber will be of the order of few microns.

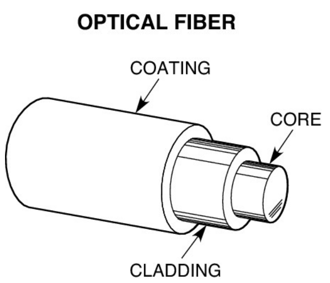


Figure 6.1: Optical fiber construction

6.2 Total Internal Reflection

Consider a ray of light moving from a denser medium to rarer medium. As a result the incident ray of light bends away from the normal. Hence the angle of refraction is greater than the angle of incidence. As the angle of incidence increases the angle of refraction also increases. For a particular angle of incidence θ_c the refracted ray grazes the interface separating the two media. The corresponding angle of incidence θ_c is called Critical Angle. If the angle of incidence is greater than the critical angle then all the light is turned back into the same medium and is called Total Internal Reflection.

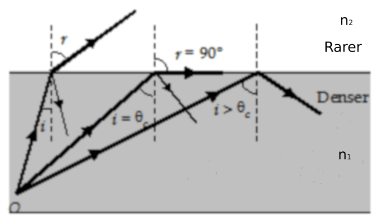


Figure 6.2: Total Internal Reflection

According to Snell's law

$$n_1 \; sin\theta_1 = n_2 \; sin\theta_2$$

$$when \; \theta_1 = \theta_c \; then, \; \theta_2 = 90^0$$

$$n_1 \; sin\theta_c = n_2 \; sin90^0$$

$$sin\theta_c = \frac{n_2}{n_1}$$

$$\theta_c = sin^{-1}\left(\frac{n_2}{n_1}\right) \tag{6.1}$$

6.3 Angle of acceptance and Numerical aperture

Acceptance angle (θ_0) is the maximum angle of incidence with which the ray is sent into the fiber core which allows the incident light to be guided by the core. It is also called as waveguide acceptance angle or acceptance cone half angle.

In optics, the numerical aperture (NA) of an optical fiber is a dimensionless number that characterizes the range of angles over which the fiber can accept light. Numerical aperture represents the light gathering capability of optical fiber and it is given by $NA = sin\theta_0$.

6.3.1 Condition for propagation

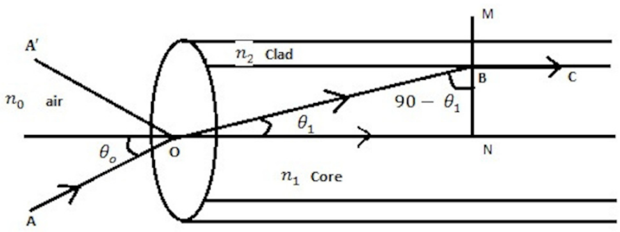


Figure 6.3: Ray propagation in the fiber

Consider an optical fiber with core made of refractive index n_1 & cladding made of material refractive index n_2. Let n_0 be the refractive index of the surrounding medium. Let a ray of light AO entering into core at an angle of incidence θ_0 w.r.t fiber axis. Then it is refracted along OB at an angle θ_1 & meet core-cladding interface at critical angle of incidence ($\theta_c = 90 - \theta_1$). Then the refracted ray grazes along BC. On applying Snell's law at O, we get

$$n_0\ sin\theta_0 = n_1\ sin\theta_1$$

$$\therefore\ sin\theta_0 = \frac{n_1}{n_0}\ sin\theta_1 \tag{6.2}$$

On applying Snell's law at point B, we get

$$n_1\ sin(90^0 - \theta_1) = n_2\ sin90^0$$

$$n_1\ cos\theta_1 = n_2$$

$$\therefore\ cos\theta_1 = \frac{n_2}{n_1} \tag{6.3}$$

From trigonometric identity

$$sin^2\theta_1 + cos^2\theta_1 = 1$$

$$sin\theta_1 = \sqrt{1 - cos^2\theta_1}$$

using equation 1.25

$$sin\theta_1 = \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2}$$

$$sin\theta_1 = \sqrt{\frac{{n_1}^2 - {n_2}^2}{{n_1}^2}}$$

$$sin\theta_1 = \frac{1}{n_1}\ \sqrt{{n_1}^2 - {n_2}^2} \tag{6.4}$$

use equation (1.26) in equation (1.24) we have,

$$sin\theta_0 = \frac{n_1}{n_0}\ \frac{1}{n_1}\ \sqrt{{n_1}^2 - {n_2}^2}$$

$$sin\theta_0 = \frac{1}{n_0}\ \sqrt{{n_1}^2 - {n_2}^2} \tag{6.5}$$

Numerical aperture $N.A = sin\theta_0$

$$N.A = \frac{1}{n_0}\ \sqrt{{n_1}^2 - {n_2}^2} \tag{6.6}$$

If the fiber is in air $n_0 = 1$ then,

$$N.A = sin\theta_0 = \sqrt{{n_1}^2 - {n_2}^2} \tag{6.7}$$

Light is transmitted through the fiber only when

$$\theta_i\ \leq\ \theta_0 \tag{6.8}$$

$$sin\theta_i\ \leq\ sin\theta_0 \tag{6.9}$$

$$sin\theta_i\ \leq\ \sqrt{{n_1}^2 - {n_2}^2} \tag{6.10}$$

$$sin\theta_i\ \leq\ N.A \tag{6.11}$$

This is the condition for propagation. Light will be transmitted through the optical fiber with multiple total internal reflections when the above condition is satisfied.

6.3.2 Fractional RI Change

Fractional index change (Δ) is defined as the ratio of difference in refractive indices of core & cladding to the refractive index of the core.

$$\Delta = \frac{n_1 - n_2}{n_1} \tag{6.12}$$

$$n_1\ \Delta = n_1 - n_2 \tag{6.13}$$

6.3.3 Relation between NA and Δ

consider the equation

$$N.A = sin\theta_0 = \sqrt{{n_1}^2 - {n_2}^2}$$

$$N.A = \sqrt{(n_1 + n_2)\ (n_1 - n_2)} \tag{6.14}$$

For small difference of n_1 & n_2, we can have,

$$n_1 \approx n_2 \tag{6.15}$$

$$\therefore\ n_1 + n_2 \approx 2n_1$$

$$\therefore\ N.A = \sqrt{(2\,n_1)\ (n_1\,\Delta)}$$

$$N.A = \sqrt{2\,{n_1}^2\,\Delta}$$

$$N.A = n_1\ \sqrt{2\,\Delta} \tag{6.16}$$

Thus the Numerical aperture can be increased by increasing the fractional index change.

6.4 Modes of propagation

Though optical fiber should support any numbers of rays for propagation practically. But it is found that the optical fiber allows only a certain restricted number of rays for propagation. The maximum number of rays or paths supported by the fiber for the propagation of light is called *Modes of propagation*.

V-number (Normalised Frequency of the fiber) An Optical fiber may be characterized by one more parameter called V-number. This determines the Number of modes supported by an optical fiber for the propagation.

$$V = \frac{\pi\, d}{\lambda}\, \sqrt{{n_1}^2 - {n_2}^2} \tag{6.17}$$

$$V = \frac{\pi\, d}{\lambda}\, N.A \tag{6.18}$$

here d is the diameter of the core, λ is wavelength, n_1 is the refractive index of the core and n_2 is the refractive index of the cladding. N.A is numerical Aperture. If the fiber is surrounded by a medium of refractive index n_0, then

$$V = \frac{\pi\, d}{\lambda}\, \frac{\sqrt{{n_1}^2 - {n_2}^2}}{n_0} \tag{6.19}$$

If $V \gg 1$, the number of modes supported by fiber can be determined using the formula

$$N \cong \frac{V^2}{2}$$

$$N \cong \frac{\pi^2\, d^2}{2\, \lambda^2} \left(\frac{{n_1}^2 - {n_2}^2}{n_0^2} \right) \tag{6.20}$$

6.4.1 Types of optical fibers

In any optical fiber, the whole material of the cladding has a uniform refractive index value. But the refractive index of the core material may either remain constant or subjected to variation in a particular pattern. The curve which represents the variation of refractive index with respect the radial distance from the axis of the fiber is called the *refractive index profile.* The optical fibers are classified under 3 categories,

1. Step index single mode fiber
2. Step index multi-mode fiber
3. Graded index multimode fiber

This classification is done depending on the refractive index profile, and the number of modes that the fiber can guide.

Step index single mode fiber

A single mode step index fiber consists of a very fine thin core of uniform RI surrounded by Cladding of RI lower than that of Core. Since there is abrupt change in the RI of Core and Cladding at the interface it is called step index fiber. Since the Core size is small the Numerical aperture is also small and hence support single mode. They accept light from LASER source. Splicing is difficult. They are used in submarine cables.

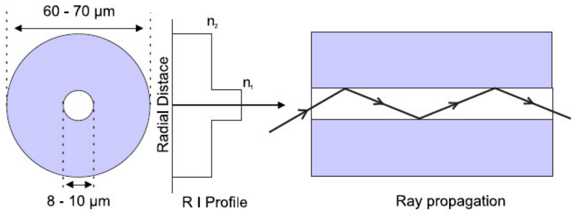


Figure 6.4: Step index single mode fiber

Step index multimode fiber

This is similar to single mode step index fiber with the exception that it has a larger core diameter. The core diameter is very large as compared to single mode optical fiber. A typical multimode step index fiber is as shown in figure. The numerical aperture is large because of large core size and thus support multimodes. They accept light from both LASER as well as from LED. They are used in data links.

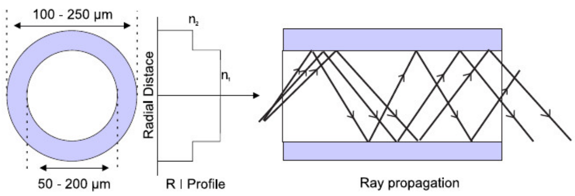


Figure 6.5: Step index multimode fiber

Graded index multimode fiber

A multimode fiber has concentric layers of RI is called GRIN fiber. It means the R I of the Core varies with distance from the fiber axis. The RI is maximum at the center and decreases with radial distance towards to core-cladding interface. The R I profile is as shown in fig. In GRIN fibers the acceptance angle and numerical aperture diminish with radial distance. The light transmission is as shown above. They accept light from both LASER as well as from LED. They are used for medium distance communication for example telephone link between central offices.

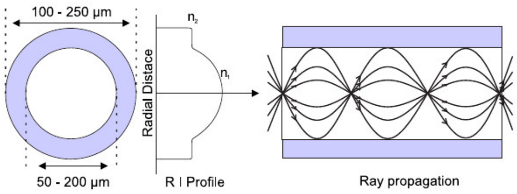


Figure 6.6: Graded index multimode fiber

6.5 Attenuation

The optical energy (signal) passing through the optical fiber gets reduced progressively. This is due to attenuation. It is also called the fiber loss or significant loss. The attenuation is measured in terms of attenuation co-efficient. The attenuation co-efficient α is defined as the ratio of optical power output to the optical power input for a fiber of length L and for a given wavelength of propagating light. It is expressed in dB/km. Attenuation co-efficient is given by

$$\alpha = \frac{-10}{L} \log_{10}\left(\frac{P_{out}}{P_{in}}\right) \quad dB/km \tag{6.21}$$

Here L is the length of the cable in km, P_{in} is Power of optical signal at launching end (input power) & P_{out} is Power of optical signal at receiving end (output power)

The attenuation in fibers gives is due to the following three losses

1. Absorption losses
2. Scattering loss (due to Rayleigh Scattering)
3. Geometric Effects (Radiation losses)

6.5.1 Absorption loss

In this type of loss, the loss of signal power occurs due to absorption of photons associated with the signal. Photons are absorbed either by impurities in the glass fiber or by pure glass material itself. Absorption loss is wavelength dependent.Thus absorption loss is classified in to two types.

Extrinsic absorption : Extrinsic loss in an optical fiber is due to the absorption of light by the impurities such as hydroxide ions and transition metal ions such as iron, chromium, cobalt and copper.

Intrinsic absorption Intrinsic loss in fiber is due to the absorption of light by the material of the fiber glass itself. The intrinsic losses are insignificant.

6.5.2 Scattering loss

Light traveling through the core can get scattered by impurities or small regions with sudden change in refractive index. Rayleigh scattering varies as $\alpha = \frac{1}{\lambda^4}$ and leads to significant power loss at smaller wavelengths. The scattering results in loss of photons. Rayleigh scattering is responsible for maximum losses in optical fibers.

6.5.3 Geometric effects

These may occur due to manufacturing defects like irregularities in fibre dimensions during drawing process or during coating, cabling or insulation processes. The microscopic bends are the bends with radii greater than fiber diameter. The microscopic bends couple light between the various guided modes of the fiber and some of them then leak through the fiber.

6.6 Applications of Optical Fibers

6.6.1 Point to point communication using Optical Fibers

In an optical fiber communication system, the input signals (audio, video or other digital data) are used to modulate light from a source like a LED or a semiconductor LASER and is transmitted through optical fiber. At the receiving end the signal is demodulated to reproduce the input signal. If data transfer takes place between only two devices then, it is called point to point communication.

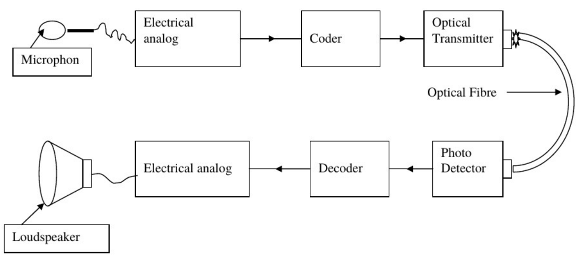


Figure 6.7: Point to point fiber optic communication system

Optical fiber communication process : The communication using Optical fiber is as follows. First voice is converted into electrical signal using a transducer. It is digitized using a Coder. The digitized signal, which carries the voice information, is fed to an optical transmitter. The light source in optical transmitter (LED or LASER Diode) emits modulated light, which is transmitted through the optical fiber. At the other end the modulated light signal is detected by a photo detector and is decoded using a decoder. Finally the information is converted into analog electrical signal and is fed to a loud speaker, which converts the signal to voice (sound).

Advantages

1. Optical fibers can carry very large amounts of information in either digital or analog form.
2. The raw material for optical fiber is of low cost and abundant.
3. It has low cost /meter/ channel
4. Cables are very compact
5. Signals are protected from radiation from lightning or sparking
6. There is no energy radiation from fiber
7. No sparks are generated

Disadvantages

1. The optical connectors are very costly
2. Maintenance cost is high
3. They cannot be bent too sharply
4. They under go structural changes with temperature

6.6.2 Fiber Optic Displacement Sensor

Principle: Light is sent through a transmitting fiber and is made to fall on a moving target. The reflected light from the target is sensed by a detector. With respect to intensity of light reflected from its displacement of the target is measured.

Construction: It consists of a bundle of transmitting fibers coupled to the laser source and a bundle of receiving fibers coupled to the detector as shown in the figure. The axis of the transmitting fiber and the receiving fiber with respect to the moving target can be adjusted to increase the sensitivity of the sensor.

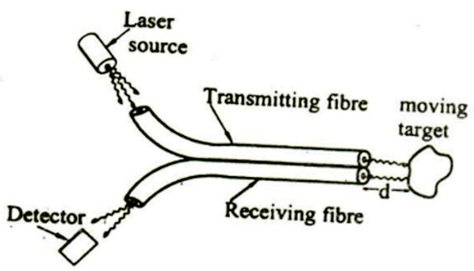


Figure 6.8: Fiber Optic Displacement Sensor

Working: Light from the source is transmitted through the transmitting fiber and is made to fall on the moving target. The light reflected from the target is made to pas through the receiving fiber and the same is detected by the detector.Based on the intensity of the light received, the displacement of the target can be measured, (i.e.) if the received intensity is more than we can say that the target is moving towards the sensor and if the intensity is less, we can say that the target is moving away from the sensor.

6.6.3 Fiber Optic Temperature Sensor based on Phase Modulation

The Mach-Zehnder Interferometric temperature sensor is as shown in the figure 6.9. This sensor offers flexible geometry and higher sensitivities. Hence it can be used for measurement of various measuranads such as temperature, pressure, rotation, strain etc. It works based on phase modulation by external measurands. Here phase of the beam through sensing fiber is compared with the reference beam. Beam splitter is used in the design of such Mach-Zehnder Interferometric sensor as shown in the figure. Beam splitter divides the light beam into two parts, one is launched into the sensing part and the other is used as reference.

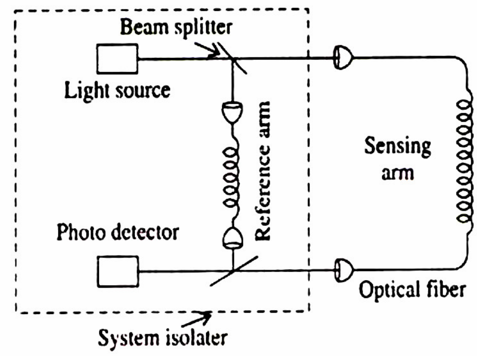


Figure 6.9: Fiber Optic Displacement Sensor

Merits and Demerits of Fiber Optic Sensors

Merits

1. It is immune from nearby EM (electromagnetic) and stray radiation.
2. It can be used in environments where high levels of electrical interference exists or where intrinsic safety is a concern.
3. It is light in weight and compact in size.
4. It is cheaper due to low manufacturing cost.
5. It offers high sensitivity and hence very small changes can also be measured.

6. It offers wide dynamic range and large bandwidth.

7. It offers multiplexing and remote sensing capabilities

8. It is tolerent against high temperature (i.e. >1450 deg centigrade) and corrosive environments.

9. It is safe and suitable to be used in extreme vibration and harse environments.

Demerits

1. It is very expensive.

2. Detection systems may be complex.

3. The users require training before they start using such sensor types.

4. Some fiber optic temperature sensors are expensive.

5. It requires precise installation methods or procedures.

6.7 Model Questions

1. Define the terms: (i) angle of acceptance, (ii) numerical aperture, (iii) fractional index change (iv) modes of propagation & (v) refractive index profile.

2. Obtain an expression for numerical aperture and arrive at the condition for propagation.

3. Give the relation between Numerical aperture and Fractional index change.

4. What is refractive index profile? Discuss different types of optical fibers with suitable diagrams.

5. What is attenuation? Explain the factors contributing to the fiber loss.

6. What is attenuation coefficient? Mention the expression for the attenuation coefficient.

7. What are the advantages of optical communications over other conventional types of communication?

8. Discuss point to point optical fiber communication system and mention its advantages over the conventional communication system.

9. Discuss the advantages and disadvantages of an optical communication.

6.8 Numerical Problems

1. Calculate the numerical aperture and angle of acceptance for an optical fiber having refractive indices 1.563 and 1.498 for core and cladding respectively.

2. The refractive indices of the core and cladding of a step index optical fiber are 1.45 and 1.4 respectively and its core diameter is $45\mu m$. Calculate its fractional refractive index change and numerical aperture.

3. Calculate numerical aperture, acceptance angle and critical angle of a fiber having a core RI 1.50 and cladding RI 1.45.

4. An optical fiber has a numerical aperture of 0.32. The refractive index of cladding is 1.48. Calculate the refractive index of the core, the acceptance angle of the fiber and the fractional index change.

5. An optical signal propagating in a fiber retains 85% of input power after traveling a distance of 500 m in the fiber. Calculate the attenuation coefficient.

6. An optical fiber has core RI 1.5 and RI of cladding is 3% less than the core index. Calculate the numerical aperture, angle of acceptance critical angle.

7. The numerical aperture of an optical fiber is 0.2 when surround by air. Determine the RI of its core, given the RI of the cladding is 1.59. Also find the acceptance angle when the fiber is in water of RI 1.33.

8. The angle of acceptance of an optical fiber is 30^0 when kept in air. Find the angle of acceptance when it is in medium of refractive index 1.33.

9. Calculate NA, V-number and number of modes in an optical fiber of core diameter $50\mu m$, core and cladding refractive indices 1.41 and 1.4 respectively at wavelength 820 nm.

10. For a step index optical fiber RI of core is 1.45 and RI of cladding is 1.40 and its core diameter is $45\mu m$. Calculate its relative refractive index difference, V-number at wavelength 1000 nm and the number of modes.

11. Calculate the number of modes of an optical fiber will transmit using the following data $n_{core} = 1.50$, $n_{clad} = 1.48$, core radius = $50\mu m$, $\lambda = 1\mu m$.

12. An optical fiber of 600 m long has input power of 120 mW which emerges out with power of 90 mW. Find attenuation in fiber.

13. The attenuation of light in an optical fiber is 3.6 dB/km. What fraction of its initial intensity is remains after i) 1 km and ii) 3 km ?

14. The attenuation of light in an optical fiber is 2.2 dB/km. What fraction of its initial intensity is remains after i) 2 km and ii) 6 km ?