APPLIED PHYSICS FOR ELECTRICAL AND ELECTRONICS STREAM

A SHORT NOTES - FIRST EDITION

DEPARTMENT OF PHYSICS
ATME COLLEGE OF ENGINEERING,
Affiliated to Visveswaraya Technological Univesrsity - Belagavi
13th km Stone, Mysuru-Bannur Road, Mysuru, PIN - 570 028.

Contents

Part I

MODULE 1 : Quantum Mechanics

Chapter 1

Quantum Mechanics

1.1 Wave-Particle dualism

The phenomena like Interference, Diffraction and Polarization are attributed to the wave properties of radiation. The Quantum theory of radiation and experiments like Photoelectric effect and Compton Effect describe the particle nature of radiation. Thus radiation behaves like waves and like particles under different suitable circumstances. Hence radiation exhibits dual nature.

1.2 de Broglie hypothesis

In the year 1924 French physicist Louis de Broglie extended wave-particle dualism through a hypothesis stating ***If radiant energy could behave like waves in some experiments and particles or photons in others and since nature loves symmetry, then one can expect the particles like protons and electrons to exhibit wave nature under suitable circumstances***. This is well known as de Broglie's hypothesis.

Therefore some sort of waves can be even associated with moving material particles called *Matter waves* or *de-Broglie waves* and the wavelength associated with matter waves is called *de Broglie wavelength*.The wavelength of a photon in-terms of its momentum is given by $\lambda = \frac{h}{p}$. Hence by analogy the de Broglie wavelength of matter waves is given by $\lambda = \frac{h}{p} = \frac{h}{mv}$. Here m is the mass of the moving particle and v is its velocity.

Note

- For a particle, charged or uncharged, moving with kinetic energy E the de Broglie wavelength is given by $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$.
- For a charged particle accelerated with a potential V volt, the de Broglie wavelength is given by $\lambda = \frac{h}{\sqrt{2mqV}}$.
- For an electron accelerated through a potential difference of V volt, the de Broglie wavelength is given by $\lambda = \frac{h}{\sqrt{2meV}}$. Further substituting the values of h, m and e, the de Broglie wavelegth is given by the expression $\lambda = \frac{12.27}{\sqrt{V}}$ Å.

1.3 Matter waves and its properties

1.3.1 Phase velocity

The velocity with which a wave travels is called phase velocity (v_p) and is also called wave velocity. If a point is marked on the wave representing the phase of the particle then the velocity with which the phase propagates from one point to another is called phase velocity.

$$v_p = \frac{\omega}{k} = \frac{E}{P} = \frac{c^2}{v} \tag{1.1}$$

Here c is the velocity of light in vacuum and v is the velocity of the matter wave. It is also evident, from the above equation 1.1, that the phase velocity is not only greater than the particle velocity it is also greater than the velocity of light. Hence there is no physical meaning for phase velocity of matter waves.

1.3.2 Wave packet and Group velocity

Since the velocity of matter waves must be equal to that of the particle velocity and since no physical meaning can be associated with phase velocity, the concept of group velocity is introduced.

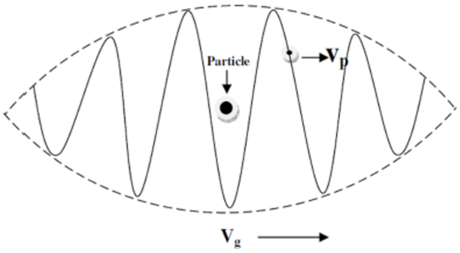


Figure 1.1: Wave group or wave packet

Matter wave can be considered as a resultant wave due to the superposition of many component waves whose velocities differ slightly. Thus a wave group or wave packet is formed. The velocity with which the wave group travels is called group velocity which is same as particle velocity. It is denoted by v_g and is as given in the equation 1.2.

$$v_g = \frac{d\omega}{dk} \tag{1.2}$$

1.3.3 Properties of matter waves

The following are the properties associated with the matter waves

1. Matter waves are associated only with particles in motion
2. They are not electromagnetic in nature
3. Group velocity is associated with matter waves
4. As a result of superposition of large number of component waves which slightly differ in frequency, matter waves are localized.
5. The phase velocity has no physical meaning for matter waves
6. The amplitude of the matter wave at a given point is associated with the probability density of finding the particle at that point.
7. The wave length of matter waves is given by $\lambda = \frac{h}{mv}$

1.4 Heisenberg's Uncertainty Principle

Statement: The simultaneous determination of the exact position and momentum of a moving particle is impossible.

Explanation : According to this principle if Δx is the error involved in the measurement of position and Δp_x is the error involved in the measurement of momentum during their simultaneous measurement, then the product of the corresponding uncertainties is given by

$$\Delta x \Delta p_x \geq \frac{h}{4\pi} \tag{1.3}$$

$$\Delta E \Delta t \geq \frac{h}{4\pi} \tag{1.4}$$

$$\Delta \theta \Delta L \geq \frac{h}{4\pi} \tag{1.5}$$

The product of the errors is of the order of Planck's constant. If one quantity is measured with high accuracy then the simultaneous measurement of the other quantity becomes less accurate.

Physical significance : According to Newtonian physics the simultaneous measurement of position and momentum are ***exact***. But the existence of matter waves induces serious problems due to the limit to accuracy associated with the simultaneous measurement. Hence the ***exactness*** in Newtonian physics is replaced by ***probability*** in quantum mechanics.

1.4.1 Application of uncertainty principle

Non-existence electrons inside the nucleus : Beta rays are emitted by the nucleus. When it was first observed it was believed that electrons exist inside the nucleus and are emitted at certain instant. If the electron can exist inside the atomic nucleus then uncertainty in its position must not exceed the diameter of the nucleus. The diameter of the nucleus is of the order of Δx_{max} is $10^{-14} m$. Applying Heisenberg's uncertainty principle for an electron expected to be inside the nucleus we get

$$\Delta x_{max} \Delta p_{min} \geq \frac{h}{4\pi} \tag{1.6}$$

$$\Delta p_{min} \geq \frac{h}{4\pi \Delta x_{max}} \tag{1.7}$$

$$\Delta p_{min} \geq \frac{6.625 \times 10^{-34}}{4 \times 3.142 \times 10^{-14}} = 5.276 \times 10^{-21} kgms^{-1} \tag{1.8}$$

Therefore, the electron should possess momentum

$$p_{min} \simeq \Delta p_{min} = 5.276 \times 10^{-21} kgms^{-1} \tag{1.9}$$

Non-relativistic equation of energy of the electron is given by

$$E = \frac{(p_{min})^2}{2m_e} = 1.53 \times 10^{-11} J \tag{1.10}$$

here m_e is the rest mass of the electron

$$E_{min} = \frac{1.53 \times 10^{-11}}{1.6 \times 10^{-19}} = 95\mathbf{MeV} \tag{1.11}$$

Conclusion : *According to experiments, the energy associated with the beta ray (electron) emission is around 3 MeV which is much lesser than the energy of the electron expected to be inside the nucleus 95 MeV. Hence electrons do not exist inside the nucleus.*

Note : Equations 1.4 and 1.5 represent the uncertainty relationship between the conjugate physical quantities (Energy,time) and (Angular displacement, Angular momentum).

1.5 Principle of Complementarity

Statement Bohr stated as *"In a situation where the wave aspect of a system is revealed, its particle aspect is concealed; and, in a situation where the particle aspect is revealed, its wave aspect is concealed. Revealing both simultaneously is impossible; the wave and particle aspects are complementary."*

Explanation We know that the consequence of the uncertainty principle is both the Wave and particle nature of the matter can not be measured simultaneously. In other words, we can not precisely describe the dual nature of Light.

- If an experiment is designed to measure the particle nature of the matter,during this experiment, errors of measurement of both position and the time coordinates must be zero and hence the momentum, energy and the wave nature of the matter are completely unknown.
- Similarly, if an experiment is designed for measuring the wave nature of the particle, then the errors in the measurement of the energy and the momentum will be zero, whereas the position and the time coordinates of the matter will be completely unknown. unknown.

From the above explanation, we can conclude that, when the particle nature of the matter is measured or displayed, the wave nature of the matter is necessarily suppressed and vice versa.

1.6 Wave Function

According to the de Broglie's hypothesis the relation between momentum and wavelength is found to be experimentally valid for both photons and particles. The quanta of matter or radiation can be represented in agreement with uncertainty principle by wave packets. Thus it suggests that concentrated bunches of waves might be used to describe localized particles and quanta of radiation. ***The variations of which make up the matter wave is called wave function***. Hence we shall consider a wave function that depends on space (x, y and z) and time(t) and is denoted by $\psi(r,t)$. The wave function for a wave packet moving along +ve x axis is given by

$$\psi = \psi_0 \ e^{i(kx-\omega t)} \tag{1.12}$$

The quantity ψ is assumed to have the following three basic properties

1. It can interfere with itself so that it can account for diffraction experiments.
2. It is large in magnitude where is particle or photon is likely to be found and small else where.
3. It will be regarded as describing the behavior of single particle or photon and not statistical distribution of number of quanta.

1.7 Time Independent Schrödinger Wave Equation

The wave equation which has variations only with respect to position and describes the steady state is called Time Independent Schrodinger wave equation. Consider a particle of mass m moving with velocity v along +ve x–axis. The de Broglie wave length λ is given by

$$\lambda = \frac{h}{mv} \tag{1.13}$$

The wave equation for one dimensional propagation of waves is given by

$$\frac{\partial^2\psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \tag{1.14}$$

The wave function is given by

$$\psi = \psi_0 \ e^{i(kx-\omega t)} \tag{1.15}$$

here ψ_0 is the amplitude at the point of consideration, ω is angular frequency and k is the wave number. Differentiating ψ twice with respect to t, we get

$$\frac{\partial^2\psi}{\partial t^2} = -\omega^2\psi_0 \ e^{i(kx-\omega t)} \tag{1.16}$$

$$\frac{\partial^2\psi}{\partial t^2} = -\omega^2\psi \tag{1.17}$$

substituting equation 1.17 in equation 1.14

$$\frac{\partial^2\psi}{\partial x^2} = \frac{1}{v^2}(-\omega^2\psi) \tag{1.18}$$

substituting for ω and v we get

$$\frac{\partial^2\psi}{\partial x^2} = \frac{1}{(f\lambda)^2}(-(2\pi f)^2\psi) \tag{1.19}$$

$$\frac{\partial^2\psi}{\partial x^2} = -\frac{4\pi^2}{\lambda^2}\psi \tag{1.20}$$

substituting for λ from equation 1.13

$$\frac{\partial^2\psi}{\partial x^2} = -\frac{4\pi^2}{(\frac{h}{mv})^2}\psi \tag{1.21}$$

$$\frac{\partial^2\psi}{\partial x^2} = -\frac{4\pi^2(mv)^2}{h^2}\psi \tag{1.22}$$

$$\frac{\partial^2\psi}{\partial x^2} = -\frac{8\pi^2 m(\frac{1}{2}mv^2)}{h^2}\psi \tag{1.23}$$

$$\frac{\partial^2\psi}{\partial x^2} = -\frac{8\pi^2 m(E-U)}{h^2}\psi \tag{1.24}$$

here

$$\frac{1}{2}mv^2 = E - U \tag{1.25}$$

here E is the total and U is potential energy of the particle.

$$\frac{\partial^2\psi}{\partial x^2} + \frac{8\pi^2 m(E-U)}{h^2}\psi = 0 \tag{1.26}$$

This can be extended to three dimension

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} + \frac{8\pi^2 m(E-U)}{h^2}\psi = 0 \tag{1.27}$$

$$\nabla^2\psi + \frac{8\pi^2 m(E-U)}{h^2}\psi = 0 \tag{1.28}$$

here the operator ∇^2 is given by

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \tag{1.29}$$

Hence the Time Independent Schrodinger equation.

1.8 Physical significance of Wave Function : Physical Interpretation

The wave function ψ just as itself has no direct physical meaning. It is more difficult to give a physical interpretation to the amplitude of the wave. The amplitude of the wave function ψ is certainly not like displacement in water wave or the pressure wave nor the waves in stretched string. It is a very different kind of wave. The quantity squared absolute value of the amplitude gives the probability of finding the particle per unit volume at the given location in space and is referred to as probability density. This is also referred to as *Born interpretation.* It is given by

$$P(x) = |\psi|^2 \tag{1.30}$$

Thus, in one dimension the probability of finding a particle in the width dx of length x

$$P(x)dx = |\psi|^2 dx \tag{1.31}$$

Similarly, in three dimension, the probability of finding a particle in a given small volume dV of volume V is given by

$$P\ \ dv = |\psi|^2 dv \tag{1.32}$$

here $dv = dx\ dy\ dz$ and P is the probability of finding the particle at given location per unit volume and is called *Probability Density*.Since ψ is a complex quantity $|\psi|^2 = \psi\psi^*$ and the product is a real number. ψ^* is the complex conjugate of ψ.

1.9 Expectation Value

In quantum mechanics ***The expectation value is the probabilistic expected value of the result (measurement) of an experiment. It can be thought of as an average of all the possible outcomes of a measurement as weighted by their likelihood.*** Expectation value as such it is not the most probable value of a measurement. In the real sense the expectation value may have zero probability of occurring. Let us consider a particle moving along the x axis.

The result of a measurement of the position x is a continuous random variable. Consider a wave function $\psi(x,t)$. The $|\psi(x,t)|^2$ value is a probability density for the position observable and $|\psi(x,t)|^2 dx$ is the probability of finding the particle between x and $x + dx$ at time t. Thus, if a measurement of position is repeated many times in an identical way on an identical particle in identical circumstances, many possible outcomes are possible and the expectation value of these outcomes is, according to the following equation

$$\langle x\rangle = \int_{-\infty}^{+\infty} x\ |\psi(x,t)|^2 dx \tag{1.33}$$

1.10 Eigen values and eigen functions

The Schrodinger wave equation is a second order differential equation. Thus solving the Schrodinger wave equation to a particular system we get many expressions for wave function (ψ). However,all wave functions are not acceptable. Only those wave functions which satisfy certain conditions are acceptable. Such wave functions are called ***Eigen functions*** for the system. The energy values corresponding to the Eigen functions are called ***Eigen values***. The wave functions are acceptable if they satisfy the following conditions.

1. ψ must be finite everywhere (Cannot be infinite)
2. ψ must be single valued which implies that solution is unique for a given position in space.
3. ψ and its first derivatives with respect to its variables must be continuous everywhere.

1.11 Applications of schrödinger wave equation

1.11.1 Particle in one-dimensional potential well of infinite height

Consider a particle of mass m bouncing back and forth between the walls of one dimensional potential well as shown in figure 1.2. The particle is said to be under bound state. Let the motion of the particle be confined along the x-axis in between two infinitely hard walls at $x = 0$ and $x = a$. Since the walls are infinitely hard, no energy is lost by the particle during the collision with walls and the total energy remains constant.

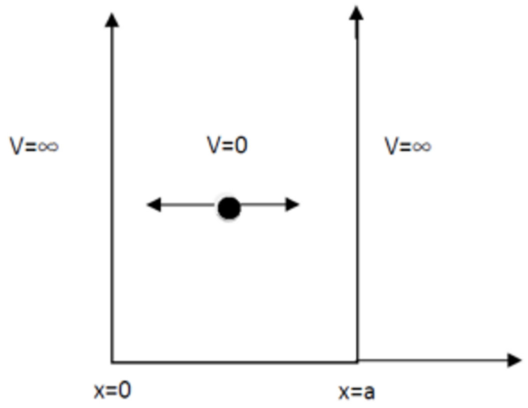


Figure 1.2: One dimensional potential well of infinite height

The description of the potential well is as follows. In between walls i.e. $0 < x < a$, the potential $V = 0$. Beyond the walls i.e. $x \leq 0$and $x \geq a$, the potential $V = \infty$.

Beyond the walls or outside the potential well

Since the particle is unable to penetrate the hard walls it exists only inside the potential well. Hence $\psi = 0$ and the probability of finding the particle outside the potential well is also zero.

In between the walls or inside the potential well

The Schrodinger wave equation is given by

$$\frac{\partial^2\psi}{\partial x^2} + \frac{8\pi^2 m(E-U)}{h^2}\psi = 0 \tag{1.34}$$

Since the potential inside the well $V = 0$, hence potential energy $U = 0$, the Schrodinger wave equation becomes

$$\frac{\partial^2\psi}{\partial x^2} + \frac{8\pi^2 m(E-0)}{h^2}\psi = 0 \tag{1.35}$$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{8\pi^2 mE}{h^2}\psi = 0 \tag{1.36}$$

$$\frac{\partial^2\psi}{\partial x^2} + k^2\psi = 0 \tag{1.37}$$

$$k^2 = \frac{8\pi^2 mE}{h^2} \tag{1.38}$$

here k is a constant for a given value of energy E. The general solution for equation 1.37 is given by

$$\psi(x) = A\sin kx + B\cos kx \tag{1.39}$$

Here in the above equation A and B are arbitrary constants which can be evaluated by applying boundary conditions.

Applying Boundary Conditions

1. The first boundary condition is, at $x = 0$, $\psi(x) = 0$. Applying this to equation 1.39, we get $0 = A\sin 0 + B\cos 0 \implies B = 0$ hence equation 1.39 reduces to

$$\psi(x) = A\sin kx \tag{1.40}$$

2. The second boundary condition is,at $x = a$, $\psi(x) = 0$. Applying this to equation 1.40, we get $0 = A\sin ka$. Since $A \neq 0$ then $\sin ka = 0$. This results in $ka = n\pi$ which further could be written as $k = \frac{n\pi}{a}$. n can take integer values. Hence equation 1.40 could be written as

$$\psi(x) = A\sin\left(\frac{n\pi x}{a}\right). \tag{1.41}$$

also from equation 1.38

$$k^2 = \frac{n^2\pi^2}{a^2} = \frac{8\pi^2 mE_n}{h^2} \tag{1.42}$$

$$E_n = \frac{n^2h^2}{8ma^2} \tag{1.43}$$

Quantization of Energy States

Substituting for $n = 1, 2, 3, 4, ..$ in the above equation ***Energy Eigen Values*** are obtained. The lowest energy state corresponds to lowest integral value of $n = 1$ which is also called as ***Zero Point Energy*** is given by $E_1 = \frac{h^2}{8ma^2}$. The energy values of a bound particle in one dimensional potential well are quantized (discrete) and are represented by the equation $E_n = n^2E_1$.

Normalization of wave function

The wave function for a particle in one dimensional potential well of infinite height is given by the equation 1.41 $\psi(x) = A\sin\left(\frac{n\pi x}{a}\right)$. In this equation A is an arbitrary constant and it can take any value. The process of determination of value of the arbitrary constant is called *Normalization of wave function*. The particle has to exist some where inside the potential the probability of the finding the particle inside the potential well is given by

$$\int_0^a |\psi(x)|^2 dx = \int_0^a P dx = 1 \tag{1.44}$$

Substituting for the wave function in the integral

$$\int_0^a A^2 \sin^2\left(\frac{n\pi x}{a}\right) dx = 1 \tag{1.45}$$

from trigonometry $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$. There fore the above equation could be written as

$$\int_0^a \frac{A^2}{2}\left[1 - \cos\left(\frac{2n\pi x}{a}\right)\right] dx = 1 \tag{1.46}$$

integrating the above equation we get

$$\frac{A^2}{2}\left[x - \frac{a}{2n\pi}\ \sin\left(\frac{2n\pi x}{a}\right)\right]_0^a = 1 \tag{1.47}$$

$$\implies A = \sqrt{\frac{2}{a}} \tag{1.48}$$

Substituting this in equation 1.41 the normalized wave function or eigen function for a particle in one dimensional potential well of infinite height is given by

$$\psi(x) = \sqrt{\frac{2}{a}}\ \sin\left(\frac{n\pi x}{a}\right). \tag{1.49}$$

The wave functions and the probability densities for the first three values of n are as shown in fig 1.3

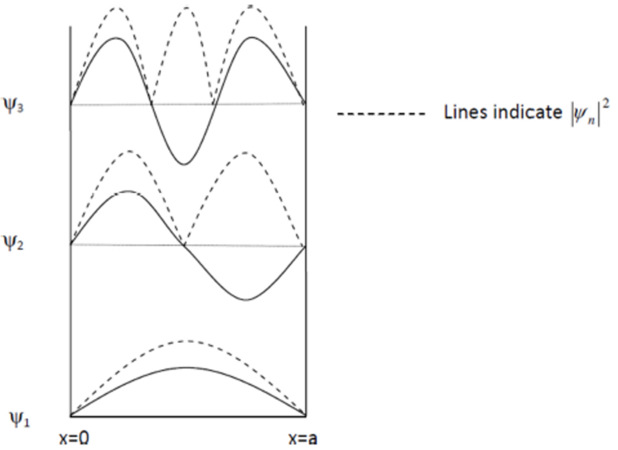


Figure 1.3: Wave function and Probability density for n = 1,2,3

Thus for ground state $(n = 1)$. The probability of finding the particle at the walls is zero and at the center $\frac{a}{2}$ is maximum. The first excited state has three nodes and the second excited state has four nodes.

Summary

Black Body radiation spectrum posed challenges during the early 19th century. Many experiments were conducted and theories were put forward to understand the phenomenon. Wein and Rayleigh-Jeans were able to explain the black-body radiation spectrum in the lower and higher wavelength region respectively. Planck's radiation law, based on discrete energy packet 'Quanta' or 'Photon', is able to explain the energy distribution in the black body radiation spectrum completely. Several experiments and theories were put forward to understand the particle and wave nature of radiations. This proved dual nature of radiation. Further, based on analogy, Louis de Broglie extended the same concept to moving particles and proposed a hypothesis which indicates the wave nature of moving particles. Diffraction experiments prove the wave nature of moving particles. Attempts were successful in knowing the different quantities related to the wave associated with particles called matter waves. Irwin Schrodinger set up a differential equation for the moving particle implementing de Broglie hypothesis. The Schrodinger wave equation is set up for a bound particle and free particles. Subsequent solutions prove that the energies of bound particles are quantized and that of free particles is continuous.

1.12 Model Questions

1. State and Explain de Broglie hypothesis.
2. What are matter waves and mention the properties.
3. Derive the relation between Phase velocity and Group velocity.
4. Derive the relation between Group velocity and Particle velocity.
5. State and Explain Heisenberg's uncertainty principle. Mention its physical significance.
6. Explain why electron cannot exist inside the nucleus?
7. State and Explain the Principle of Complementarity.
8. Define wave function? Mention its basic properties.
9. Derive Time independent Schrodinger wave equation for a particle moving in three dimension.
10. Discuss the physical interpretation of wave function?
11. Elucidate expectation value.
12. Describe Eigen functions and Eigen values.
13. Discuss the motion of a particle in one dimensional potential well of infinite height.
14. Discuss the energies of a free particle using time independent schrodinger equation.

1.13 Numerical Problems

1. Calculate the momentum of the particle and the de Broglie wavelength associated with an electron with a kinetic energy of $1.5keV$.
2. Calculate the wavelength associated with an electron having kinetic energy $100eV$.
3. Calculate de Broglie wavelength associated with electron carrying energy $2000eV$.
4. Find the energy of the neutron in eV whose de Brogle wavelength is 1 Å.
5. Calculate de Broglie wavelength associated with neutron of mass 1.674×10^{-27kg} with $\frac{1}{10}^{th}$ part of speed of light.
6. Calculate de Broglie wavelength associated with electrons whose speed is 0.01 part of the speed of light.
7. What is the de Broglie wavelength of a proton whose energy is $3eV$ given mass of proton is $1.67\times10^{-27}kg$.
8. Find the kinetic energy and group velocity of an electron with de Broglie wavelength of $0.2nm$.
9. Calculate the de Broglie wavelength of particle of mass $0.65\frac{MeV}{c^2}$ has a kinetic energy $80eV$.
10. Find de Broglie wavelength of a particle of mass $0.58\frac{MeV}{c^2}$ has a kinetic energy $90eV$, Where c is speed of light.
11. A particle of mass $940\frac{MeV}{c^2}$ has kinetic energy $0.5keV$. Find it de Broglie wavelength, where c is speed of light.
12. Find the de Broglie wavelength of an electron accelerated through a potential difference of $182V$ and object of mass $1kg$ moving with a speed of $1ms^{-1}$. Compare the results and comment.
13. The position and momentum of an electron with energy $0.5keV$ are determined. What is the minimum percentage uncertainty in its momentum if the uncertainty in the measurement of position is 0.5Å.
14. The speed of electron is measured to with in an uncertainty of $2.2 \times 10^4 ms^{-1}$ in one dimension. What is the minimum width required by the electron to be confined in an atom?
15. Estimate the time spent by an atom in the excited state during the excitation and de-excitation processes, when a spectral line of wavelength $546nm$ and width $10^{-14}m$ is emitted.
16. An electron is confined to a box of length $10^{-9}m$, calculate the minimum uncertainty in its velocity.
17. The position and momentum of 1 keV electron are simultaneously determined. If it position is located within 1Å, find the uncertainty in the determination of its momentum.
18. A spectral line of wavelength 4000 Å has a width of 8×10^{-5}Å. Evaluate the minimum time spent by the electrons in the upper energy state between the excitation and de-excitation processes.
19. The inherent uncertainty in the measurement of time spent by Iridium 191 nuclei in the excited state is found to be $1.4 \times 10^{-10}s$. Estimate the uncertainty that results in its energy in eV in the excited state.
20. An electron is bound in one dimensional potential well of width $0.18nm$. Find the energy value in eV of the second excited state.
21. The first excited state energy of an electron in an infinite well is $240eV$. What will be its ground state energy when the width of the potential well is doubled?
22. A quantum particle confined to one–dimensional box of width a is in its first excited state. What is the probability of finding the particle over an interval of $\frac{a}{2}$ marked symmetrically at the center of the box.

Part II

MODULE 2 : Electrical Properties of Materials

Chapter 2

Conductors

2.1 Quantum free electron theory of metals

The failures of classical free electron theory led to the rise of Quantum Free electron theory and was proposed by Sommerfield in the year 1928. The quantum free electron theory is based on the following assumptions.

2.1.1 Assumptions

1. Unlike classical free electron theory, in quantum free electron theory, energy values of free electrons are quantized.
2. Thus in a metal there exists large number of closely spaced energy levels for free electrons which form a band.
3. The distribution of free electrons in the energy levels occur as per Pauli's exclusion principle. Only maximum of two electrons can occupy a given an energy level.
4. The potential setup by the lattice ions is assumed to be constant throughout the metal.
5. The mutual repulsion between electrons and the attraction between electrons and lattice ions are neglected.

2.1.2 Fermi energy

Consider a metal containing *N* number of atoms. Thus there exists *N* number of energy levels in each band. These energy levels are very closely spaced. The filling up of energy levels in bands occur as per Pauli's exclusion principle and thus each level can accommodate a pair of electrons with spin up and spin down. Thus electrons in a metal start filling up the available energy levels from the lower most level of the lower most band. All bands below the valence band are completely filled. The valence band is partially filled. **The highest filled energy level in a metal at absolute zero by free electrons is called *Fermi Level* and the corresponding energy is called *Fermi Energy* (E_F).** Thus, at absolute zero and with no electric field applied, all levels below Fermi level are completely filled and above Fermi level are empty.

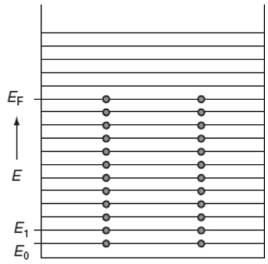


Figure 2.1: Energy Band diagram and Fermi Energy

2.1.3 Density of States (DoS)

According to band theory of solids the number of energy levels in a band is equal to the number of atoms present in the solid. Out of all bands the valence band is of most concern. The spacing between any two energy levels in an energy band is not a constant quantity. The density of levels in a band increases with increase in energy hence it depends on the energy value itself. Each level can accommodate two electrons with opposite spins. Hence an energy level supports two states for electron occupation.

The Density of States is defined as the number of energy states available per unit volume of the material in the unit energy range in the valence band of the material. It is mathematically a continuous function denoted by $g(E)$. Consider a very small energy range dE centered around E such that the density around E remains constant. The number of energy levels in the energy range E and $E + dE$ per unit volume of the material is given by $g(E)dE$. The variation of $g(E)dE$ as a function of E is given by

$$g(E)dE = \left(\frac{8\sqrt{2}\pi m^{3/2}}{h^3}\right) E^{\frac{1}{2}} dE \tag{2.1}$$

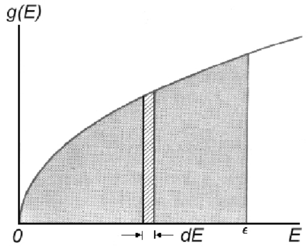


Figure 2.2: Density of states function vs Energy

2.1.4 Fermi–Dirac Statistics and Fermi factor

a metal contains large number free of electrons. The occupation of energy levels in the valence band is according to Pauli's exclusion principle. This distribution of electrons follows a certain universal rule called Fermi-Dirac Distribution. This statistics is applicable for the distribution of particles of spin half. A distribution statistics provides the probability of occupation of an energy level with energy (E) at a given temperature T under thermal equilibrium. Thus the probability of occupation of an energy level of energy (E) at temperature (T) under thermal equilibrium is given by *Fermi Factor* $(f(E))$.

$$f(E) = \frac{1}{e^{\left(\frac{E-E_f}{kT}\right)} + 1} \tag{2.2}$$

The number density of electrons defined as the number of free electrons per unit volume the material in the energy range E and $E + dE$ in the valence band of the material is given by $n(E)dE$. Mathematically

$$N(E)dE = g(E)dEf(E) \tag{2.3}$$

Here $f(E)$ is the Fermi factor.

The number density of electrons is the product of the number of available energy states and their occupation probability. This is called Fermi-Dirac distribution. As per this rule at temperature absolute zero all levels below the Fermi level are completely filled and above the Fermi level are empty. At higher temperatures the probability of occupation of energy levels above the Fermi level increases and below the Fermi level decreases due to the increase in thermal energy of the valence electrons.

2.1.5 Dependence of Fermi factor on energy and temperature

As described, the Fermi factor is a function of energy and temperature. This dependence could be explained for energy levels below and above Fermi level at absolute zero and higher temperatures.

Probability of occupation of levels with energy $E < E_F$ and at T = 0K

The Fermi factor or Fermi function is given by

$$f(E) = \frac{1}{e^{\left(\frac{E-E_f}{kT}\right)} + 1} \tag{2.4}$$

Here $E - E_f$ is negative. Substituting the value for $T = 0$

$$f(E) = \frac{1}{e^{\left(\frac{E-E_f}{k*0}\right)} + 1}$$

$$f(E) = \frac{1}{e^{-\infty} + 1} = \frac{1}{0 + 1}$$

There fore $f(E) = 1$. Hence, at $T = 0K$, all energy levels below the Fermi level are completely filled.

Probability of occupation of levels with energy $E > E_F$ and at T = 0K

The Fermi factor or fermi function is given by

$$f(E) = \frac{1}{e^{\left(\frac{E-E_f}{kT}\right)} + 1} \tag{2.5}$$

Here $E - E_f$ is positive. Substituting the value for $T = 0$

$$f(E) = \frac{1}{e^{\left(\frac{E-E_f}{k*0}\right)} + 1}$$

$$f(E) = \frac{1}{e^{\infty} + 1} = \frac{1}{\infty + 1}$$

There fore $f(E) = 0$. Hence, at $T = 0K$, all energy levels above the Fermi level are empty.

Probability of occupation of levels with energy $E = E_F$ and at T > 0K

The Fermi factor or Fermi function is given by

$$f(E) = \frac{1}{e^{\left(\frac{E-E_f}{kT}\right)} + 1} \tag{2.6}$$

Here $E - E_f = 0$. Substituting the values

$$f(E) = \frac{1}{e^{\left(\frac{0}{kT}\right)} + 1} \tag{2.7}$$

$$f(E) = \frac{1}{e^{\left(\frac{0}{kT}\right)} + 1} \tag{2.8}$$

$$f(E) = \frac{1}{1 + 1} = \frac{1}{2} = 0.5 \tag{2.9}$$

Thus for all temperatures above 0 K the probability of occupation of Fermi level is ½. Thus the variation of Fermi factor with temperature is as shown in the graph 2.3

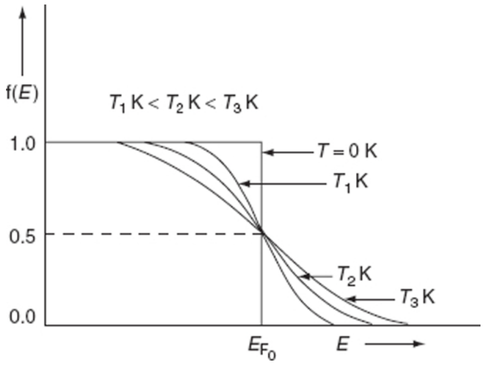


Figure 2.3: Variation of $f(E)$ as a function of Temperature and Energy

2.1.6 Variation of number density of e^- with temperature - Fermi Dirac Distribution

As per Fermi-Dirac statistics the number density of electrons in the energy range E and $E + dE$ is given by the product of density of states and the Fermi factor. Therefore

$$N(E)dE = g(E)dEf(E) \tag{2.10}$$

Graphically The distribution of free electrons in a metal at various temperatures and energy levels is as shown in the graph 2.4.

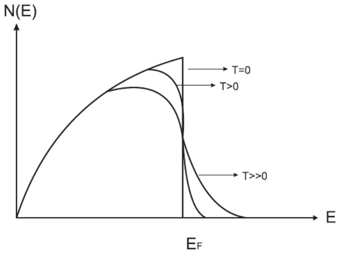


Figure 2.4: Fermi-Dirac Distribution of Conduction Electrons

Fermi Temperature

It is the temperature at which the average thermal energy of the free electron in a metal is equal to Fermi energy at 0 K. The thermal energy of the electron is given by $E = kT$, here k is Boltzmann constant. Hence Fermi temperature is given by the equation.

$$T_F = \frac{E_F}{k} \tag{2.11}$$

The Fermi energy of copper is $7eV$. The calculation of Fermi temperature for copper is as given below

$$T_F = \frac{7 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23}}$$

$$T_F = 81159K$$

This is temperature cannot be realized in practice. Thus the Fermi temperature is a theoretical concept.

Effective mass of free electron

Free electrons in a metal will be in drift motion due to the applied electric field. Thus the motion of free electrons in a metal is actually influenced by applied electric field and the periodic potential well. Hence mass of the electron is different from its rest mass and is called *Effective mass*. The effective mass of the electron is denoted by m^*.

2.1.7 Expression for electrical conductivity using QFET

$$\sigma = \frac{ne^2\tau_F}{m^*} \tag{2.12}$$

Is the equation for electrical conductivity as per quantum fre electron theory.

Model Questions

1. Give the assumptions on which quantum free electron theory is based/
2. Describe Fermi level and Fermi energy.
3. Explain the density of states and mention density of states function.
4. Write a note of Fermi function and Fermi-Dirac distribution statistics.
5. Explain the variation of Fermi factor with energy and temperature.
6. Write a note on Fermi velocity, Fermi temperature and effective mass. Mention the expression for electrical conductivity derived using Quantum free electron theory of metals.
7. Derive an expression for Fermi energy at 0K using quantum free electron theory of metals.
8. Describe the success of quantum free electron theory of metals.

Numerical Problems

1. Calculate the Fermi velocity and the mean free path for conduction electrons in silver, given that its Fermi energy is $5.5eV$, and the relaxation time for electrons is $3.83 \times 10^{-14}s$.
2. The Fermi level in silver is $5.5eV$ at $0\ K$. Calculate the number of free electrons per unit volume and the probability of occupation for electrons with energy $5.6eV$ in silver at the same temperature.
3. Calculate the free electron concentration in aluminium metal assuming that each of its atom contributes 3 free electrons for conduction. Given, for Aluminium, resistivity = $2.7 \times 10^{8}\Omega m$, atomic weight = 26.98 and density = $2.7 \times 10^{3}kgm^{-3}$.
4. Calculate the Fermi energy in eV for a metal at Zero kelvin, whose density is $10500kgm^{-3}$, atomic weight is 107.9 and it has one conduction electron per per atom. Given $N_A = 6.025 \times 10^{26}kmole^{-1}$.
5. Calculate the probability of an electron occupying an energy level $0.02eV$ above the Fermi level at $200K$ and $400K$.
6. Calculate the probability of an electron occupying an energy level $0.02eV$ below the Fermi level at $400K$.
7. The Fermi level for silver is $5.5eV$. What is the energy for which probability of occupancy at $300K$ is 0.01
8. Find the temperature at which there is 1% probability that a state with an energy 0.5eV above Fermi energy is occupied.
9. Find the free electron density in a metal with Fermi energy 3 eV.
10. Calculate the Fermi energy (in eV), Fermi velocity and Fermi temperature for sodium, given that the density of sodium is $971kgm^{-3}$. Its atomic weight is 23, and it has one conduction electron per atom.
11. Find the probability that an energy level at $0.2eV$ below Fermi level being occupied at temperature $300K$ and $1000K$
12. The Fermi level in Pottasium is 2.1 eV at a particular temperature. Calculate the number of free electrons per unit volume in pottasium at the same temperature.
13. Calculate the probability that an electron occupies an energy level 0.02 eV above Fermi level at 300 K.
14. In an energy band of a material of $1cm^3$ volume, Calculate the Density of States at $2.5eV$ above the bottom of the band. Ans.$1.07 \times 10^{22}cm^{-3}eV^{-1}$.

Chapter 3

Dielectric Constant

Faraday discovered that the capacitance of the condenser increases when the region between the plates is filled with dielectric. If C_0 is the capacitance of the capacitor without dielectric and C is the capacitance of the capacitor with dielectric then the ratio $\frac{C}{C_0}$ givesϵ_r called relative permittivity or Dielectric constant. Also for a given isotropic material the electric flux density is related to the applied field strength by the equation $D = \epsilon E$, Here ϵ is Absolute permittivity. In SI system of units the relative permittivity is given by the ratio of absolute permittivity to permittivity of free space.

$$\epsilon = \epsilon_0 \epsilon_r \tag{3.1}$$

Here ϵ_0 is permittivity of free space. ϵ_r is relative permittivity or dielectric constant.Dielectric constant is also denoted by K.

For an isotropic material, under static field conditions, the relative permittivity is called static dielectric constant. It depends on the structure of the atoms of which the material is composed.

3.0.1 Polarization of Dielectrics

The displacement of charged particles in atoms or molecules of dielectric material so that net dipole moment is developed in the material along the applied field direction is called polarization of dielectric Polarization is measured as net dipole moment per unit volume and is a vector quantity.

$$P = N\mu \tag{3.2}$$

Here P is polarization, N number of dipoles per unit volume, μ is average dipole moment per molecule. Also μ is given by

$$\mu = \alpha E \tag{3.3}$$

Here α is defined as Polarizability.

3.0.2 Relation between Dielectric constant and Polarization

As the polarization P measures the additional flux density arising from the presence of the material as compared to free space it has the same unit as D and is related to it as

$$\vec{P} = \epsilon_0 \left(\epsilon_r - 1\right) \vec{E} \tag{3.4}$$

3.0.3 Types of Polarization or Polarization mechanisms

The electrical polarization takes place through four different mechanisms. They are

1. Electronic polarization P_e
2. Ionic polarization P_i
3. Orientation polarization P_o
4. Space charge polarization P_s

The net polarization of the material is due to the contribution of all four polarization mechanism

$$P = P_e + P_i + P_o + P_s \tag{3.5}$$

Electronic polarization

This involves the separation of the center of the electron cloud around an atom with respect to the center of its nucleus under the application of electric field. Hence dipoles are induced within the material. This leads to the development of net dipole moment in the material and is the vector sum of dipole moments of individual dipoles.

Figure 3.1: Electronic Polarization

Ionic polarization

This occurs in ionic solids such as sodium chloride etc. Ionic solids possess net dipole moment even in the absence of external electric field. But when the external electric field is applied the separation between the ions further increases. Hence the net dipole moment of the material also increases

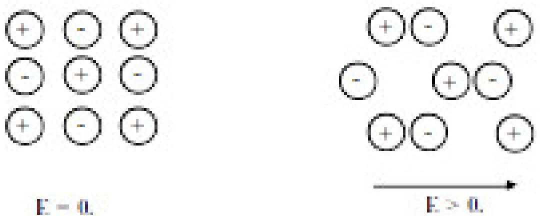


Figure 3.2: Ionic Polarization

Orientation polarization

This occurs in polar dielectric material, which possesses permanent electric dipoles. In polar dielectrics the dipoles are randomly oriented due thermal agitation. Therefore net dipole moment of the material is zero. But when the external electric field is applied all dipoles tend to align in the field direction. There fore dipole moment develops across the material. This is referred to as orientation polarization.It is temperature dependent.

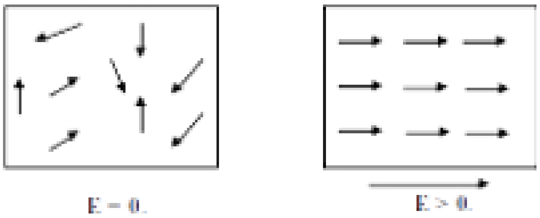


Figure 3.3: Orientation Polarization

Space charge polarization

This involves limited movement of charges resulting in alignment of charge dipoles under applied field. This usually happens at the grain boundaries or lattice defects and localized charge is set up. Since this is very small it can be neglected.

3.0.4 Classification of Dielectrics

Polar Dielectrics

Polar dielectric materials possess permanent electric dipoles in the material and are oriented in random directions so that the net dipole moment of the material is zero in the absence of applied electric field. If polar dielectric materials are placed in the external electric field then all dipoles tend to align in the field direction and hence net dipole moment develops cross dielectric material. This is the polarization of polar dielectric materials. Materials like H_2O, NH_3 are the examples for polar dielectrics.

Non Polar Dielectrics

Non polar dielectric materials do not possess permanent electric dipoles. Thus the net dipole moment across the material is zero in the absence of external electric field. In non polar dielectric materials dipoles are induced due to the applied electric field which results in the net dipole moment in the dielectric material in the direction of the applied field. This is the polarization of non-polar dielectric materials.Elementary gasses like He, H_2 are the examples for non polar dielectrics.

3.1 Internal fields in Dielectrics

3.1.1 Definition

When a dielectric material is placed in the external electric field polarization occurs. Hence the net electric field at any point within the dielectric material is given by The sum of external field and the field due to all dipoles surrounding that point. This net field is called internal field or Local field.

3.1.2 Expression for Internal field in case of one dimension

Consider a dielectric material placed in the external electric field E. Consider an array of dipoles in the polarized dielectric material as shown in the figure 3.4. Let a be the distance between to successive dipoles in the array.

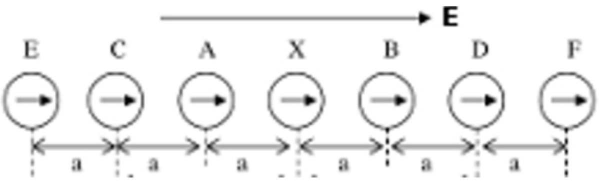


Figure 3.4: One Dimensional Array of Dipoles

the internal field at a dipole 'X'due to all dipoles in the array is given by

$$E_i = E + \frac{1.2\mu}{\pi\epsilon_0 a^3} \tag{3.6}$$

Here mu is the dipole moment, ϵ_0 is the permittivity of free space, a is the inter dipole distance in the array.

$$E_i = E + \frac{1.2\alpha E}{\pi\epsilon_0 a^3} \tag{3.7}$$

here α is polarizability. Since α,ϵ_0 and a are positive quantities the local field $E_i > E$.

3.1.3 Expression for Internal field in case of three dimension- Lorentz Field

For three dimension $\frac{1}{a^3}$ could be replaced with number of dipoles per unit volume N and $\frac{1.2}{\pi}$ could be replaced with γ, in equation 3.7. Thus we get

$$E_i = E + \frac{\gamma N \alpha E}{\epsilon_0} \tag{3.8}$$

$$E_i = E + \frac{\gamma P}{\epsilon_0} \tag{3.9}$$

Here the polarization $P = N\alpha E$.

For an elemental solid dielectric material $\gamma = \frac{1}{3}$, Thus equation 3.9 becomes

$$E_L = E + \frac{P}{3\epsilon_0} \tag{3.10}$$

Thus the Lorentz field is given by equation 3.10.

3.1.4 Derivation of Clausius - Mossotti equation

Consider an Elemental solid dielectric material. Since they don't posses permanent dipoles, for such materials, the ionic and orientation polarizabilities are zero. Hence the polarization P is given by

$$P = N\alpha_e E_L \tag{3.11}$$

Here $E_L = E + \frac{P}{3\epsilon_0}$ is the Lorentz field. Substituting for Lorentz Field in 3.11, we get

$$P = N\alpha_e \left[E + \frac{P}{3\epsilon_0} \right]$$

$$P = N\alpha_e E + N\alpha_e \frac{P}{3\epsilon_0}$$

$$P - N\alpha_e \frac{P}{3\epsilon_0} = N\alpha_e E$$

$$P \left[1 - \frac{N\alpha_e}{3\epsilon_0} \right] = N\alpha_e E$$

$$P = \frac{N\alpha_e E}{\left[1 - \frac{N\alpha_e}{3\epsilon_0} \right]} \tag{3.12}$$

The relation between the Polarization and Electric field strength is given by

$$P = \epsilon_0 (\epsilon_r - 1) E \tag{3.13}$$

equating equations 3.12 and 3.13

$$\frac{N\alpha_e E}{\left[1 - \frac{N\alpha_e}{3\epsilon_0} \right]} = \epsilon_0 (\epsilon_r - 1) E$$

$$\frac{N\alpha_e}{\epsilon_0 (\epsilon_r - 1)} = \left[1 - \frac{N\alpha_e}{3\epsilon_0} \right]$$

$$\frac{N\alpha_e}{\epsilon_0 (\epsilon_r - 1)} + \frac{N\alpha_e}{3\epsilon_0} = 1$$

$$\frac{N\alpha_e}{3\epsilon_0} \left[\frac{3}{(\epsilon_r - 1)} + 1 \right] = 1$$

$$\frac{N\alpha_e}{3\epsilon_0} \left[\frac{\epsilon_r + 2}{\epsilon_r - 1} \right] = 1$$

$$\frac{N\alpha_e}{3\epsilon_0} = \frac{\epsilon_r - 1}{\epsilon_r + 2} \tag{3.14}$$

Equation 3.14 is called Clausius-Mossotti relation.

Numerical Problems

1. Determine the polarization produced in a crystal by an electric field of strength 6000 V/cm if it has a dielectric constant of 5. Ans. $2.124 \times 10^{-5} Cm^{-2}$
2. Calculate the dielectric constant of NaCl crystal if the polarization developed is $4.3 \times 10-8$ when subjected to electric field of $2000Vm^{-1}$. Ans. 3.428.
3. An elemental solid dielectric material has polarizability $7 \times 10^{-40} Fm^{-2}$. Assuming the internal field to be Lorentz, calculate the dielectric constant for the material if the material has $3 \times 10^{28} atoms/m^3$. Ans. 12.33.
4. The dielectric constant of sulphur is 3.4. Assuming a cubic lattice for its structure, calculate the electronic polarizability of sulphur. Given: for sulphur density = 2.07 gm/cc, and atomic weight 32.07. Ans. $\alpha_e = 3.035 \times 10^{-40} Fm^2$.
5. The atomic weight and density of sulphur are 32 and $2.08 \times 10^3 kg/m^3$. The electronic polarizability of the atom is $3.28 \times 10^{-40} F/m^2$. If Sulphur solid has a cubic structure, calculate its dielectric constant. Ans. 3.873.
6. A solid contains $5 \times 10^{28} atoms/m^3$ each with a polarisability of $2 \times 10^{-40} Fm^2$. Assuming that the internal field is given by Lorentz formula. Calculate the ratio of internal field to the external field. Given $\epsilon_0 = 8.854 \times 10^{-12} Fm^{-1}$.

Chapter 4

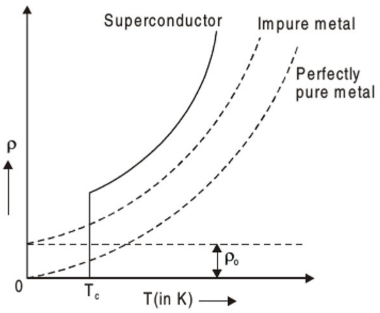
Superconductivity

4.1 Introduction to Superconductivity

Discovery : Lord Kamerlingh Onnes discovered the phenomenon of superconductivity in the year 1911. When he was studying the temperature dependence of resistance of Mercury at very low temperature he found that resistance of Mercury decreases with temperature with the decrease in temperature up to a particular temperature T_c = 4.15K . Below this temperature the resistance of mercury abruptly drops to zero. Between $4.15K$ and $0K$ Mercury offered no resistance for the flow of electric current. This phenomenon is reversible and material becomes normal once again when temperature was increased above 4.15K. This phenomenon is called superconductivity and material which exhibits the property is named superconductor.

Definition : Thus the Superconductivity is defined as ***"The phenomenon in which resistance of certain metals, alloys and compounds drops to zero abruptly, below certain temperature is called superconductivity***

Variation of Resistivity with Temperature The variation of the resistivity of a superconductor,pure and impure metals with temperature is as shown below.

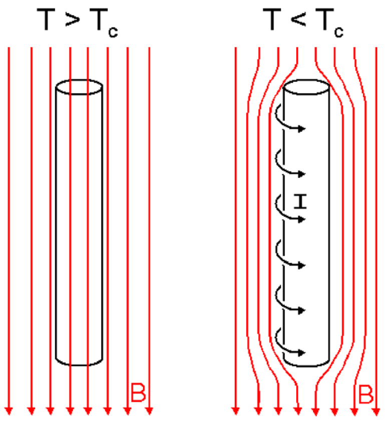


Critical Temperature : The temperature, below which materials exhibit superconducting property is called critical temperature, denoted by T_c. Critical temperature T_c is different for different substances. The materials, which exhibit superconducting property, are called superconductors.

Above critical temperature material is said to be in normal state and offers resistance for the flow of electric current. Below critical temperature material is said to be in superconducting state. Thus T_c is also called as transition temperature.

4.2 Meissner's Effect

In 1933, Meissner and Ochsenfeld showed that when a superconducting material is placed in a magnetic field it allows magnetic lines of force to pass through, if it's temperature is above T_c. If the temperature is reduced below the critical temperature Tc then it expels all the flux lines completely out of the specimen and exhibits perfect diamagnetism. This is known as Meissner's effect. Since superconductor exhibits perfect diamagnetism below the critical temperature Tc, magnetic flux density inside the material is zero.



The expression for magnetic flux density is given by

$$B = \mu_0(M + H) \tag{4.1}$$

Here B is Magnetic Flux Density, M is Magnetization and H is the applied magnetic field strength. For a superconductor, $B = 0$ at $T < T_c$. Thus we get

$$M = -H \tag{4.2}$$

Thus Meissner's Effect signifies the negative magnetic moment associated with superconductors.

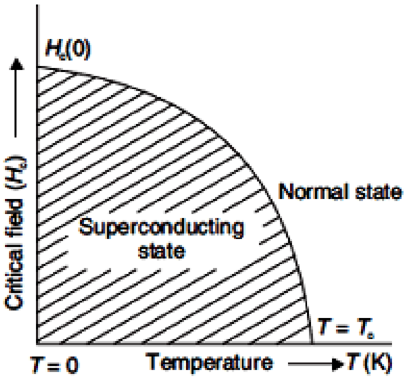
4.3 Critical Field and its Temperature Dependence

Critical field We know that when superconductor is placed in a magnetic field it expels magnetic flux lines completely out of the body and exhibits a perfect diamagnetism. But if the strength of the magnetic field is further increased, it is found that for a particular value of the magnetic field, material looses its superconducting property and becomes a normal conductor. The value of the magnetic field at which the transition occurs from the Superconducting state to Normal Conducting state is called **Critical Field or Critical Magnetic Field** and is denoted by H_c . It is found that by reducing the temperature of the material further superconducting property of the material could be restored. Thus, critical field does not destroy the superconducting property of the material completely but only reduces the critical temperature of the material.

The variation of Critical field with temperature below the critical temperature is given by

$$H_c = H_0 \left[1 - \frac{T^2}{T_c^2} \right] \tag{4.3}$$

Here H_c is the Critical field at any temperature T less than T_c, H_0 is the Critical field at $T = 0K$.



4.4 Types of Superconductors

Superconductors are classified into two types

1. Type I Superconductor or Soft Superconductor
2. Type II Superconductor or Hard Superconductor

Type I Superconductors

Type I superconductors exhibit complete Meissener's Effect and posses a single value of critical field . The graph of magnetic moment Vs magnetic field is as shown in the Fig.4.1. As the field strength increases the material becomes more and more diamagnetic until H becomes equal to H_c. Above H_c the material allows the flux lines to pass through and exhibits normal conductivity. The value of H_c is very small for soft superconductors. Therefore soft superconductors cannot withstand high magnetic fields. Therefore they cannot be used for making superconducting magnets. Ex. Hg, Pb and Zn.

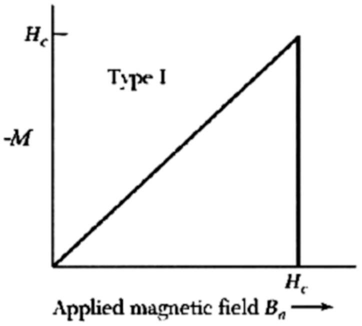


Figure 4.1: Type1 Superconductor

Type II Superconductors

paragraph Superconducting materials, which can withstand high value of critical magnetic fields, are called Hard Superconductors.

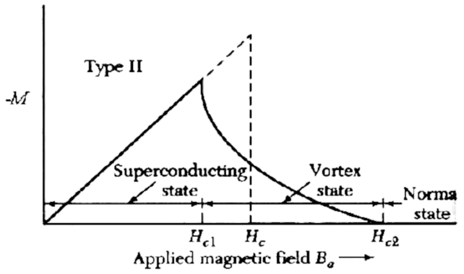
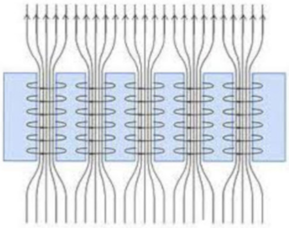


Figure 4.2: Type2 Superconductor

The graph of magnetic moment Vs magnetic field is as shown in the Fig.4.2. Hard superconductors are characterized by two critical fields H_{c1} and H_{c2} . When applied magnetic field is less than H_{c1} material exhibits perfect diamagnetism. Beyond H_{c1} partial flux penetrates and the material is said to be **Vortex State**. Thus flux penetration occurs through small-channelized regions called filaments. As the strength of the field increases further, more and more flux fills the body and thereby decreasing the diamagnetic property of the material. At H_{c2} flux fills the body completely and material losses its diamagnetic property as well as superconducting property completely.



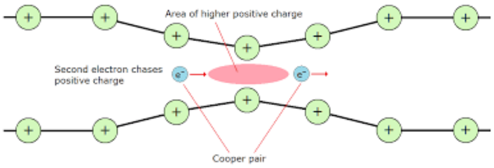
The value of H_{c2} is hundreds of times greater than Hc of soft superconductors. Therefore they are used for making powerful superconducting magnets. Examples:$NbTi$, Nb_3Sn

4.5 BCS Theory of Superconductivity

Bardeen, Cooper and Schrieffer explained the phenomenon of superconductivity in the year 1957. The essence of the BCS theory is as follows.

Consider an electron approaching a positive ion core and suffers attractive coulomb interaction. Due to this attraction ion core is set in motion and thus distorts that lattice. Let a second electron come in the way of distorted lattice and interaction between the two occurs which lowers the energy of the second electron. The two electrons therefore interact indirectly through the lattice distortion or the phonon field which lowers the energy of the electrons. The above interaction is interpreted as electron - Lattice - electron interaction through phonon field.

It was shown by Cooper that, this attractive force becomes maximum if two electrons have opposite spins and momentum. The attractive force may exceed coulombs repulsive force between the two electrons below the critical temperature, which results in the formation of bound pair of electrons called cooper pairs.

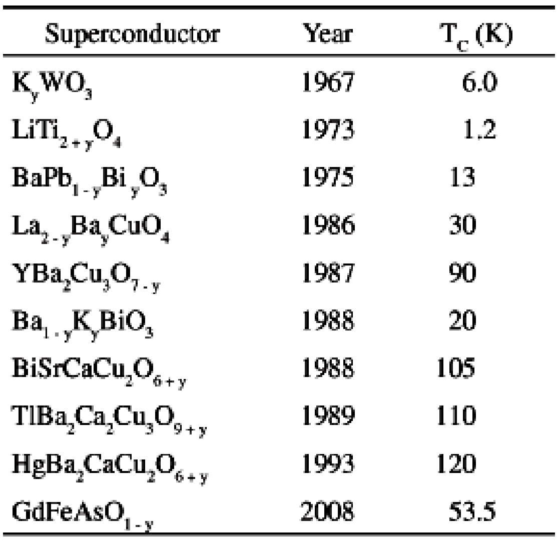


Below the critical temperature the dense cloud of Cooper pairs form a collective state and the motion all Cooper pairs is correlated resulting in zero resistance of the material.

4.6 High Temperature Superconductivity

Superconducting materials which exhibit superconductivity at relatively higher temperatures are called high temperature superconductors. Thus high temperature superconductors posses higher value of critical temperature compared to conventional superconductors. Most of the high temperature superconductors are found to fall into the category of ceramics. In 1986 George Bednorz and Alex Muller discovered a compound containing Lanthanum, Barium, Copper and Oxygen having T_c =30K was developed. In 1987 scientists developed a compound which is an oxide of the form $YBa_2Cu_3O_7$ which is referred to as 1-2-3 compound with $T_c > 90K$ was discovered.

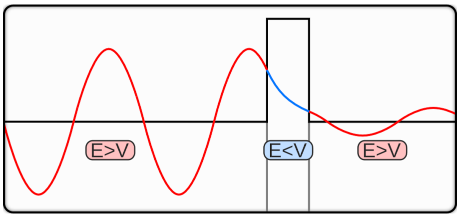
All high temperature superconductors are oxides of copper and bear Perovskite crystal structure characterized by large number of copper-oxygen layers. It was found that addition of extra copper-oxygen layer pushes the critical temperature T_c to higher values. The super currents are strong in the copper-oxygen layer and weak in the direction perpendicular to the planes. Following is the list of High Temperature Superconductors.



Superconductor	Year	T_c (K)
K_yWO_3	1967	6.0
$LiTi_{2+y}O_4$	1973	1.2
$BaPb_{1-y}Bi_yO_3$	1975	13
$La_{2-y}Ba_yCuO_4$	1986	30
$YBa_2Cu_3O_{7-y}$	1987	90
$Ba_{1-y}K_yBiO_3$	1988	20
$BiSrCaCu_2O_{6+y}$	1988	105
$TlBa_2Ca_2Cu_3O_{9+y}$	1989	110
$HgBa_2CaCu_2O_{6+y}$	1993	120
$GdFeAsO_{1-y}$	2008	53.5

4.7 Quantum Tunneling

In classical mechanics, when a particle has insufficient energy, it would not be able to overcome a potential barrier. In the quantum world the particles can often behave like waves. On encountering a barrier, a quantum wave will not end abruptly. Rather its amplitude decrease exponentially. This drop in amplitude corresponds to a drop in the probability of finding a particle further into the barrier. If the barrier is thin enough, then the amplitude may be non-zero on the other side. This would imply that there is a finite probability that some of the particles will tunnel through the barrier.



In regions where the potential energy is higher than the wave's energy, the amplitude of the wave decays exponentially. If the region is narrow enough, the wave can have a non-zero amplitude on the other side.

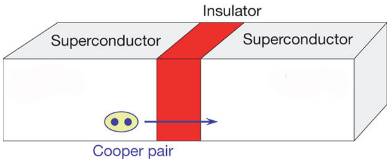
4.8 AC and DC Josephson Junctions

4.8.1 Josephson Junction

In 1962, Brian Josephson predicted that Cooper pairs could tunnel through a very thin insulating layer separating two super conductors. The superconductor-insulator-superconductor layer constitutes the Josephson junction.

4.8.2 DC Josephson Effects

Consider a Josephson junction consisting of two superconducting metal films separated by a thin oxide barrier of 10 to 20 Å thick.The Cooper pairs tunnel from one side of the junction to the other side easily. This due to the phase difference introduced by the insulator between the wave function of Cooper pairs on both sides the junction. Due to the phase difference a super current flows through the junction even if the applied voltage is zero. This is known as the **DC Josephson effect**. Josephson showed that the super current through the junction is given by



$$I_S = I_C sin\phi_0 \tag{4.4}$$

Here ϕ_0 is the phase difference between the wave functions describing Cooper pairs on both sides of the barrier, and I_C is the critical current at zero voltage condition. I_C depends on the thickness and width of the insulating layer and the temperature.

4.8.3 AC Josephson Effect

If we apply a dc voltage across the Josephson junction, it introduces an additional phase on Cooper pairs during tunneling. As a result a strikingly new phenomenon will be observed. The dc voltage generates an alternating current I given by

$$I_S = I_C sin(\phi_0 + \Delta\phi) \tag{4.5}$$

Because of the dc voltage V applied across the barrier, the energies of Cooper pairs on both sides of the barrier differ in energy by 2eV.

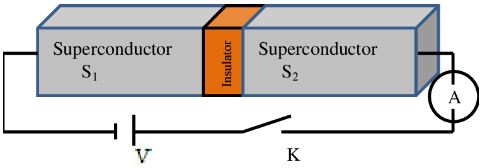


Figure 4.3: DC and AC Josephson Effects

4.9 DC and RF Squids

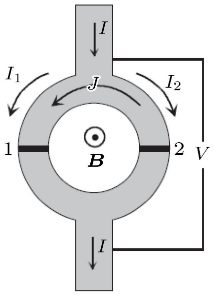
4.9.1 SQUID

A superconducting quantum interference device (SQUID) is a device used to measure extremely weak magnetic flux. Thus, it is basically a sensitive magnetometer made of a superconducting ring. The flux penetrating a superconducting loop is quantized in steps of $\phi_0 = \frac{h}{2e}$. The heart of a SQUID is a superconducting ring, which contains one or more Josephson junctions. There are two main types of SQUID: DC SQUID and RF (or AC) SQUID.

4.9.2 DC Squid

The DC SQUID was invented in 1964 by Robert Jaklevic, John Lambe, Arnold Silver, and James Mercereau. It has two Josephson junctions in parallel in a superconducting loop. It is based on the DC Josephson effect. It relies on the interference of currents form each junction.

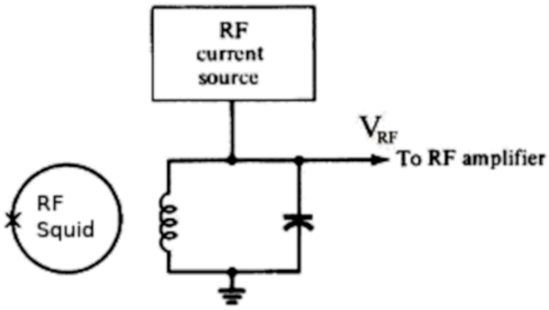
The dc SQUID is biased with a dc current equal to about twice the superconducting current I_c. Dc voltage across the junctions is created. Change in the flux penetrating the loop enhances the current through one JJ and reduces the current through the other. This leads to JJs working asymmetrically, one JJ is driven normally and one is superconducting. This asymmetry provides a feedback current that nulls the flux penetrating the SQUID loop. Total flux

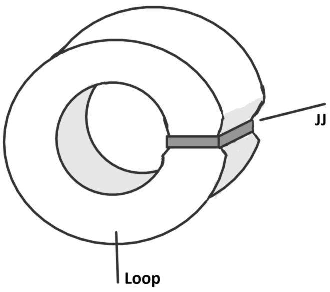


within the loop is multiples of ϕ_0. The feedback current is a direct measure of changes in flux applied to the SQUID.

4.9.3 RF (AC) Squid

The RF SQUID was invented in 1965 by Robert Jaklevic, John J. Lambe, Arnold Silver, and James Edward Zimmerman at Ford. It is based on the AC Josephson effect and uses only one Josephson junction. It is less sensitive compared to DC SQUID but is cheaper and easier to manufacture in smaller quantities.





In RF SQUID the flux is coupled into a loop containing a single JJ through an input coil and an RF coil. RF coil is part of a high-Q resonant circuit to read out current changes due to induced flux in the SQUID loop. The tuned circuit is driven by a constant RF oscillator which is weakly coupled to the loop. Measuring the change in the input coil current is done by counting the number of periods the coil produces in the detected RF output, because the detected RF output is a periodic function.

Note : The DC SQUIDs offer higher sensitivity, but RF SQUIDs have lower sensitivity. RF SQUIDs are commonly used form of the sensor, because of their ease and low price of manufacturing in small batches.

4.10 Model Questions

1. Define Phonon. Describe Mathiessen's rule.
2. Explain the failures of Classical Free Electron Theory of Metals.
3. Mention the Assumptions of Classical Free electron theory of metals.
4. Explain the concept of Fermi Level, Fermi Energy, Density of States.
5. Define Fermi Factor. Discuss the variation of Fermi Factor with Energy and Temperature and represent graphically.
6. Discuss the discovery of Superconductivity and hence discuss the variation of resistivity with temperature in superconductor with critical temperature as reference.
7. State and explain Meissner's Effect.
8. Define Critical field and hence explain its variation with temperature below critical temperature.
9. Distinguish between/Explain Type-1 and Type-2 superconductors.
10. Describe the evolution of high temperature of superconductivity.
11. Elucidate the BCS Theory of Superconductivity.
12. Explain the phenomenon of quantum tunneling.
13. Define a Josephson Junction and hence explain the DC and AC Josephson effects.
14. Define Squid and describe DC and RF Squids.
15. Brief the applications of superconductivity in quantum computing.

4.11 Numerical Problems

1. Lead has superconducting transition temperature of $7.26K$. If the initial field at $0K$ is $50 \times 10^3 Am^{-1}$ Calculate the critical field at $6K$.
2. A superconducting tin has a critical temperature of $3.7K$ at zero magnetic field and a critical field of 0.0306 tesla at $0K$. Find the critical field at $2K$.

3. The superconducting transition temperature of Lead is $7.26K$. Calculate the initial field at $0K$ given the critical field at 5 K as $33.644 \times 10^3 Am^{-1}$
4. Calculate the ratio of critical fields for a superconductor at $7K$ and $5K$ give the critical temperature $8K$.
5. The critical field for niobium is $1 \times 10^5 Am^{-1}$ at $8K$ and $2 \times 10^5 Am^{-1}$ at $0K$. Calculate the transition temperature of the element.

Part III

MODULE 3 : LASER AND OPTICAL FIBERS

Chapter 5

LASER

5.1 Introduction

LASER is an acronym for Light Amplification by Stimulated Emission of Radiation. The first LASER was built by Theodore H Maiman in the year 1960. Thus it finds various applications starting from industries to communication.

5.2 Characteristics of a LASER beam

The LASER beam has the following four characteristics.

1. LASER beam is highly monochromatic.
2. LASER beam is highly coherent.
3. LASER beam is highly directional.
4. LASER is a high intensity beam of light.

5.3 Interaction of radiation with matter

The interaction between radiation and matter occurs through the following three processes.

1. Induced absorption
2. Spontaneous emission
3. Stimulated emission

5.3.1 Induced Absorption

When a photon of right energy is incident on the atom then the photon is absorbed This process is induced by the photon and hence it is called Induced Absorption.

Consider an atom in a lower energy states E_1, it will excite to higher energy states E_2 by absorbing the incident photon of energy $E = h\nu = E_2 - E_1$. Here E_1 energy of the lower energy state, E_2 is the energy of the higher energy state, h is the Planck's constant ν is the frequency of photon.

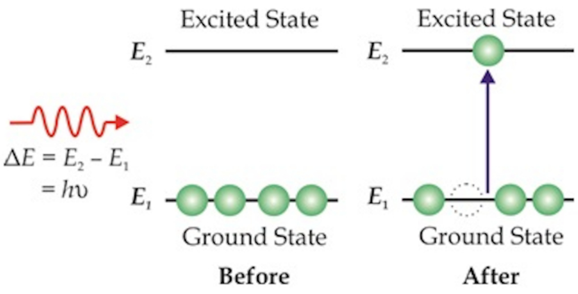


Figure 5.1: Induced absorption

5.3.2 Spontaneous Emission

Spontaneous emission is the process of emission of photon, when an atom transits from higher energy level to lower energy level without the influence of any external energy.

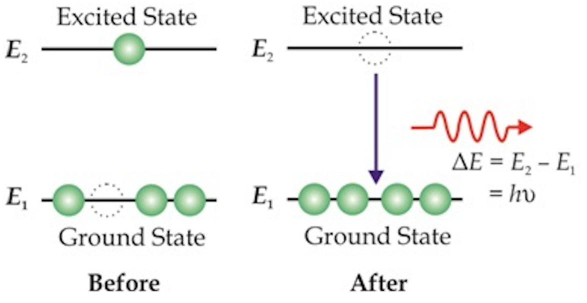


Figure 5.2: Spontaneous emission

An electron in the higher energy state of an atom makes a transition to lower energy state without the action of any external agency. the energy of the photon emitted is given by $E = h\nu = E_2 - E_1$. In this process the emitted photons need not travel in the same direction. Thus the light beam is not directional.

5.3.3 Stimulated Emission

When a photon of suitable energy interacts with an atom in the higher energy state then the atom is stimulated (Forced) to make transition from higher energy state to a lower energy state with the emission of a photon. Both the incident photon and the emitted photons are coherent and travel in the same direction. Thus the process is called stimulated emission.

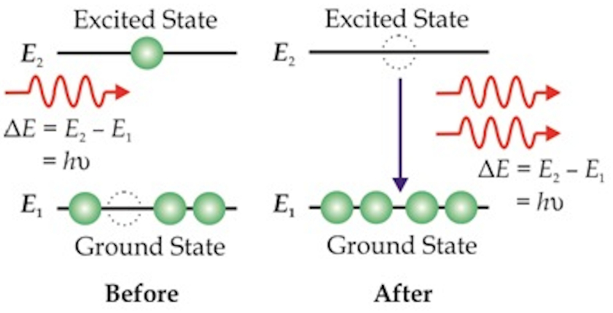


Figure 5.3: Stimulated emission

When a photon of energy $h\nu = E_2 - E_1$ interacts with an atom in the higher energy state the stimulated emission takes place with the emission of two photons of same energy that are highly directional and coherent.Thus stimulated emission could be used to generate a highly coherent directional beam of light.

5.4 Einstein's A and B co-efficients and expression for energy density

Consider a system containing N atoms and is under thermal equilibrium. Let E_1 and E_2 be the lower and higher energy levels that contain N_1 and N_2 number of atoms respectively. Let the incident energy density of the radiation be E_ν. Hence the system absorbs and emits the energy through the following processes. The energy of the photons absorbed and emitted by the atoms is $E = h\nu = (E_2 - E_1)$

Rate of induced absorption

The rate of induced absorption is defined as the number of induced absorption per second per unit volume in unit time. Rate of absorption depends on

1. Number of atoms in the lower energy state N_1.
2. The incident energy density E_ν.

Hence

1. Rate of Induced absorption $\propto N_1 E_\nu$
2. Rate of Induced absorption $= B_{12} N_1 E_\nu$

Here B_{12} is proportionality constant called Einsteins coefficient of Induced absorption.

Rate of spontaneous emission:

The number of spontaneous emission per unit volume in unit time is called rate of spontaneous emission. Rate of spontaneous emission depends on

Since spontaneous emission is a voluntary process it is independent of energy density E_ν. The rate of spontaneous emission depends only on the number of atoms in the higher energy state N_2. Thus

1. Rate of spontaneous emission $\propto N_2$
2. Rate of Spontaneous emission $= A_{21} N_2$

Here A_{21} is the proportionality constant called Einstein's co-efficient of spontaneous emission.

Rate of stimulated emission

The number of stimulated emission per unit volume in unit time is called rate of stimulated emission. Rate of stimulated emission depends upon,

1. Number of atoms in the higher energy state (N_2)
2. The energy density (E_ν).

Hence

1. The Rate of stimulated emission $\propto N_2 E_\nu$
2. Rate of stimulated emission $= B_{21} N_2 E_\nu$

Here the proportionality constant called B_{21} is Einstein's coefficient of stimulated emission.

Under Thermal Equilibrium the total Energy of the System remains unchanged. Hence Rate of Absorption is equal to rate of emission.

∴ Rate of Induced Absorption = [Rate of Spontaneous emission + Rate of Stimulated Emission]

∴

$$B_{12} N_1 E_\nu = A_{21} N_2 + B_{21} N_2 E_\nu \tag{5.1}$$

$$(B_{12} N_1 - B_{21} N_2) E_\nu = A_{21} N_2$$

$$E_\nu = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2} \tag{5.2}$$

$$E_\nu = \frac{A_{21}}{B_{12}\frac{N_1}{N_2} - B_{21}} \tag{5.3}$$

$$E_\nu = \frac{A_{21}}{B_{21}}\left[\frac{1}{\frac{B_{12}}{B_{21}}\frac{N_1}{N_2} - 1}\right] \tag{5.4}$$

According to Boltzmann relation the we have

$$\frac{N_2}{N_1} = e^{\frac{-h\nu}{kT}} \tag{5.5}$$

or we can re-write as,

$$\frac{N_1}{N_2} = e^{\frac{h\nu}{kT}} \tag{5.6}$$

Here h is the Planck's constant, c is the speed of light in vacuum, λ is the wavelength of the photon, k is the Boltzmann constant and T is the absolute temperature. Substituting for $\frac{N_1}{N_2}$ in equation 5.4

$$E_\nu = \frac{A_{21}}{B_{21}} \left[\frac{1}{\frac{B_{12}}{B_{21}} \left(e^{\frac{h\nu}{kT}} \right) - 1} \right] \tag{5.7}$$

According to Planck's radiation law, the equation for energy density in the frequency domain is given by

$$E_\nu = \frac{8\pi h\nu^3}{c^3} \left[\frac{1}{e^{\frac{h\nu}{kT}} - 1} \right] \tag{5.8}$$

on comparing equations 5.7 and 5.8 we can get

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \tag{5.9}$$

and

$$\frac{B_{12}}{B_{21}} = 1 \tag{5.10}$$

or $B_{12} = B_{21}$

This means that Probability of Induced absorption is equal to Probability of Stimulated emission. Hence A_{21} & B_{21} can be replaced by A & B. Thus equation 5.7 could be written as

$$E_\nu = \frac{A}{B} \left[\frac{1}{e^{\frac{h\nu}{kT}} - 1} \right] \tag{5.11}$$

Hence the expression for energy density in terms of Einstein's co-efficient A and B.

5.5 LASER Action and the Conditions for LASER action

Consider a LASER system. Let an atom in the excited state is stimulated by a photon of right energy so that atom makes stimulated emission. Two coherent photons are obtained. These two coherent photons if stimulate two atoms in the exited state to make emission then four coherent photons are emitted. These four coherent photons stimulate 4 more atoms in the excited state resulting in 8 coherent photons and so on. As the process continues number of coherent photons increases. These coherent photons constitute an intense beam of LASER. This phenomenon of building up of number of coherent photons so as to get an intense LASER beam is called lasing action.

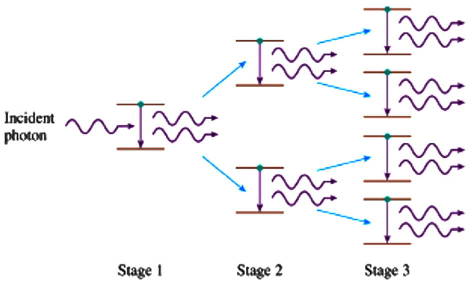


Figure 5.4: LASER action

LASER action could be achieved through the conditions population inversion and meta-stable state.

5.5.1 Population Inversion and Pumping

If a system is under thermal equilibrium the number of atoms in excited state is less than the number of atoms in the lower energy state. For the production of LASER number of stimulated emission must be more when compared to induced absorption and spontaneous emission. This is possible only if the number of atoms in the higher energy state is more than the number of atoms in the lower energy state and is called population inversion. The means of achieving population inversion by supplying energy from a suitable source is called Pumping. In addition,to have more stimulated emissions, the life time of atoms in the excited state must be longer. Thus the essential conditions for population inversion are

1. Higher energy state should posses a longer life time.
2. The number of atoms in the higher energy state must be greater than the number of atoms in the lower energy state.

5.5.2 meta-stable state

The life time of an energy level is of the order of 10^{-8} second. If an atom posses unusual longer life time in an energy state such a state is referred to as a meta-stable state. Usually the life time of meta-stable state varies from 10^{-2}s to 10^{-3}s. Population inversion could be achieved with the help of three energy state with one of them a meta-stable state and is as shown in the figure 5.5. The population inversion is achieved between the state E_2 and E_1 as state E_2 is a meta-stable state.

Note : The principles of Laser are

1. Stimulated Emission
2. Population Inversion
3. meta-stable State

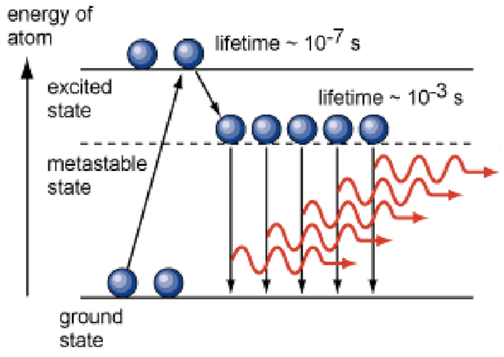


Figure 5.5: Population inversion, E_2 is meta-stable state

5.6 Requisites of a LASER system

The three requisites of a LASER system are,

1. Excitation source for pumping action
2. Active medium that supports meta-stable states
3. LASER cavity

5.6.1 Energy Source or Pumping Mechanism

In order to achieve population inversion more and more atoms are to be moved to higher energy state and is called pumping. This is achieved by supplying suitable energy using an energy source. If optical energy is used then the pumping is called optical pumping and if electrical energy is used then the pumping is called electrical pumping.

5.6.2 Active medium

Population inversion occurs at certain stage in the active medium due to the absorption of energy. The active medium supports meta-stable states. After this stage the active medium is capable of emitting LASER light.

5.6.3 Resonant cavity (or) LASER cavity

The LASER Cavity consists of an active medium bound between two highly parallel mirrors. The reflection of photons from the mirrors results in multiple traverse of photons through the active medium inducing more and more stimulated emissions. Thus amplification of light is achieved. This also helps to tap certain permissible part of LASER energy from the active medium.The cavity resonates and the output will be maximum when the distance L between the mirrors is equal to an integral multiple of $\frac{\lambda}{2}$. Here λ is the wavelength of incident suitable radiation.the length of the LASER cavity is expressed as

$$L = \frac{n\,\lambda}{2} \tag{5.12}$$

5.7 Construction and Working of CO_2 LASER

It is a molecular gas LASER and was designed by Dr. C.K.N. Patel of BELL labs in 1963. In molecular LASER, the lasing action occurs between vibration-rotation energy levels of the molecules. It operates in the middle IR region. it is a four level LASER producing both continuous and pulsed output wave-forms. It has very high efficiency of upto 30%. it is widely used in industry and medical applications.

CO_2 molecule has one carbon atom about which two oxygen atoms are symmetrically located. CO_2 molecule can vibrate in 3 different modes. In each mode, the centre of gravity remains same.

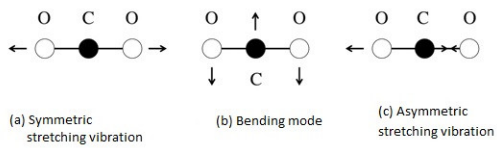


Figure 5.6: Mode of vibration

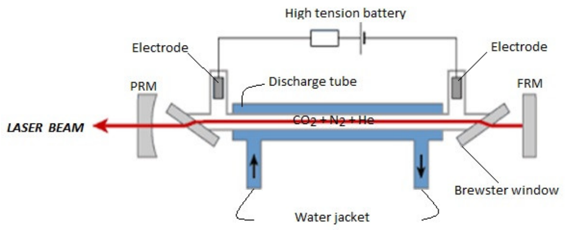


Figure 5.7: Carbon Dioxide LASER

Construction: A typical CO_2 gas LASER is as shown in figure. It consists of a long narrow discharge tube of diameter 2.5 cm and length about 5m made of fused quartz. The tube is maintained cool & is filled with active medium, a mixture of CO_2, N_2 & He gas in the ratio 1:2:3. Sometimes, water vapor is added to reoxidise. Depending on the usage either AC or DC voltage is applied between the electrode to provide Electrical discharge pumping. Brewster windows polarize the LASER beam.

Working:

When a high DC voltage is supplied across th electrodes, discharge of gases takes place. During the discharge electrons collide with N_2 and CO_2 molecules. Thus

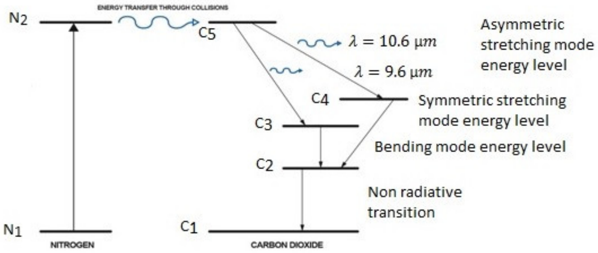


Figure 5.8: CO_2 LASER energy level diagram

the molecules absorb energy and are excited to higher energy level. This is called *Collision of First Kind.*

$$e^1 + N_2 \rightarrow N_2^* + e^2 \tag{5.13}$$

$$e^3 + CO_2 \rightarrow CO_2^* + e^4 \tag{5.14}$$

Since an higher energy level N_2 of Nitrogen molecule matches with one of the *vibration – rotation* level C_5 of CO_2 the resonance transfer of energy takes place. This is referred to as *Collision of Second Kind.*

$$N_2^* + CO_2 \rightarrow N_2 + CO_2^* \tag{5.15}$$

Due to continuous discharge population inversion is achieved between the level C_5 and C_4 also between the levels C_5 & C_3 as shown in the energy level diagram. The transition from C_5 to C_4 and C_3 results in the emission of LASER of wavelengths $10.6\mu m$ and $9.6\mu m$ respectively in the IR region of the EM spectrum. The system gets heated due to non-radiative transitions during the LASER action.

Advantages

1. It generates both continuous and pulsed LASER output.
2. It has High efficiency compared to other LASER devices

Applications:

1. It is used in industrial applications like welding, cutting, drilling , etc.
2. It is used in LIDAR due to minimum atmospheric attenuation.
3. It finds application in communication systems.

5.8 Applications of LASER

LASER has wide range of applications pertaining all disciplines of engineering. Here in the syllabus only two applications are discussed relavant to computing.

5.8.1 Application of LASER in defense - LASER Range Finder

Ranging is finding the accurate distance of location of an object from a reference place. During the war the ranging has to be done with a very high accuracy so that shell hits and destroys the enemy tank successfully. The laser beam is ideally suited due to its high intensity and high directionality, It can travel a few kilometer with very low losses in intensity and very low divergence. The LASER range finders using neodymium and carbon dioxide LASERs have become a standard item for artillery and tanks.

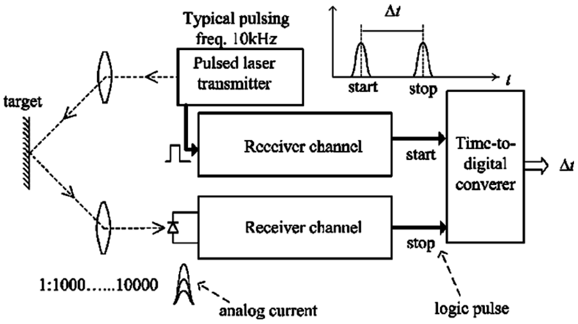


Figure 5.9: LASER Range Finder

A schematic diagram of a typical LASER range finder is as shown in the figure 5.9. The LASER range finder works on the principle of a radar and is called LIDAR which stands for Light Detection and Ranging. It makes use of the characteristic properties of the LASER beam, namely, monochromaticity, high intensity, coherency, and directionality. A collimated pulse of the LASER beam is directed towards a target and the reflected 1ight from the target is received by an optical system and detected. The time taken by the LASER beam for the to and fro travel from the transmitter to the target is measured. When half of the time thus recorded is multiplied by the velocity of light, the product gives the range.

5.8.2 Advantages

The advantages of LASER range finders over conventional range finders.

1. They are light weight.
2. They posses high reliability.
3. They possess superior range.

5.8.3 LASER Printer

Laser printers ware invented at XEROX in 1969 by by researcher Gary Starkweather.Laser Printers are digital printing devices that are used to create high quality text and graphics on plain printer. A Diode Laser is used in the process of printing in LASER Printer.

Construction

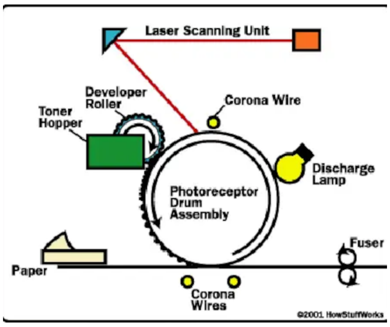


Figure 5.10: Laser Printer Schematic

Working Principle

1. A laser beam projects an image of the page to be printed onto an electrically charged rotating Photo sensitive drum coated with selenium.
2. Photo conductivity allows charge to leak away from the areas which are exposed to light and the area gets positively charged.
3. Toner particles are then electrostatically picked up by the drum's charged areas, which have been exposed to light.
4. The drum then prints the image onto paper by direct contact and heat, which fuses the ink to the paper.

Advantages

1. Laser printers are generally quiet and fast.
2. Laser printers can produce high quality output on ordinary papers.
3. The cost per page of toner cartridges is lower than other printers.

Disadvantages

1. The initial cost of laser printers can be high.
2. Laser printers are more expensive than dot-matrix printers and ink-jet printers

5.9 Model Questions

1. What is LASER? Enumerate the Characteristics of a LASER Beam.
2. Discuss the three possible ways through which radiation and matter interaction can take place.
3. Explain the terms, (i) Induced absorption, (ii) Spontaneous emission, (iii) Stimulated emission, (iv) Population inversion, (v) Meta-stable state & (vi) Resonant cavity.
4. Explain the rates of absorption and emission and hence derive an expression for energy density using Einstein's A and B coefficients.
5. Explain requisites of LASER system.
6. What is Semiconductor LASER? Describe with energy band diagram the construction & working of Semiconductor diode LASER along with applications.
7. Discuss the working of LASER barcode reader.
8. With the help of a sketch describe the principle, construction and working of the LASER Printer.
9. Explain LASER Cooling and its application.

5.10 Numerical Problems

1. Find the ratio of population of two energy levels in a LASER if the transition between them produces light of wavelength 6493 Å, assuming the ambient temperature at 27°C.
2. Find the ratio of population of two energy levels in a medium at thermal equilibrium, if the wavelength of light emitted at 291 K is 6928 Å.
3. The ratio of population of two energy levels out of which one corresponds to metastable state is 1.059×10^{-30}. Find the wavelength of light emitted at 330 K.
4. Find the ratio of population of two energy levels in a medium at thermal equilibrium, if the wavelength of light emitted at 300 K is $10\mu m$. Also find the effective temperature when energy levels are equally populated.
5. The average power output of a LASER beam of wavelength 6500 Å is 10 mW. Find the number of photons emitted per second by the LASER source.
6. The average power of a LASER beam of wavelength 6328 Å is 5 mW. Find the number of photons emitted per second by the LASER source.
7. A pulsed LASER has an average power output 1.5 mW per pulse and pulse duration is 20 ns. The number of photons emitted per pulse is estimated to be 1.047×10^{8}. Find the wavelength of the emitted LASER.
8. A pulsed LASER with power 1 mW lasts for 10 ns. If the number of photons emitted per pulse is 5×10^{7}. Calculate the wavelength of LASER.

9. A Ruby LASER emits a pulse of 20 ns duration with average power per pulse being 100 kW. If the number of photons in each pulse is 6.981×10^{15}, calculate the wavelength of photons.

10. In a LASER system when the energy difference between two energy levels is 2×10^{-19} J, the average power output of LASER beam is found to be 4 mW. Calculate number of photons emitted per second.

Chapter 6

Optical Fibers

6.1 Introduction

Optical fibers are the wires and strands made of transparent dielectrics which guide light over longer distances using the phenomenon of ***Total Internal Reflection***. Many optical fibers are bundled together and are given a protective layer of covering using an insulating material. This bundle is called Optical Fiber Cable or Fiber Bundle (Bundle Fiber).

Construction: The sectional view of a typical optical fiber is as shown in the figure. It has three regions named Core, Cladding and Sheath.

1. The innermost light guiding region is called Core.
2. The layer covering core and helps in total internal reflection of light is called Cladding or Clad.
3. The outermost protective layer is called Sheath (Coating). The sheath protects the fiber from mechanical stress and chemical reactions.

The optical fiber is designed to support total internal reflection and hence the RI of coren_1 is made greater than the RI of cladding n_2. A typical fiber will be of the order of few microns.

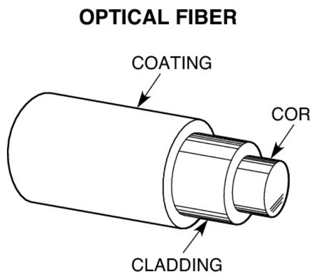


Figure 6.1: Optical fiber construction

6.2 Total Internal Reflection

Consider a ray of light moving from a denser medium to rarer medium. As a result the incident ray of light bends away from the normal. Hence the angle of refraction is greater than the angle of incidence. As the angle of incidence increases the angle of refraction also increases. For a particular angle of incidence θ_c the refracted ray grazes the interface separating the two media. The corresponding angle of incidence θ_c is called Critical Angle. If the angle of incidence is greater than the critical angle then all the light is turned back into the same medium and is called Total Internal Reflection.

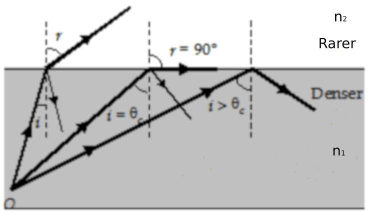


Figure 6.2: Total Internal Reflection

According to Snell's law

$$n_1\ sin\theta_1 = n_2\ sin\theta_2$$

$$when\ \theta_1 = \theta_c\ then,\ \theta_2 = 90^0$$

$$n_1\ sin\theta_c = n_2\ sin90^0$$

$$sin\theta_c = \frac{n_2}{n_1}$$

$$\theta_c = sin^{-1}\left(\frac{n_2}{n_1}\right) \tag{6.1}$$

6.3 Angle of acceptance and Numerical aperture

Acceptance angle (θ_0) is the maximum angle of incidence with which the ray is sent into the fiber core which allows the incident light to be guided by the core. It is also called as waveguide acceptance angle or acceptance cone half angle.

In optics, the numerical aperture (NA) of an optical fiber is a dimensionless number that characterizes the range of angles over which the fiber can accept light. Numerical aperture represents the light gathering capability of optical fiber and it is given by $NA = sin\theta_0$.

6.3.1 Condition for propagation

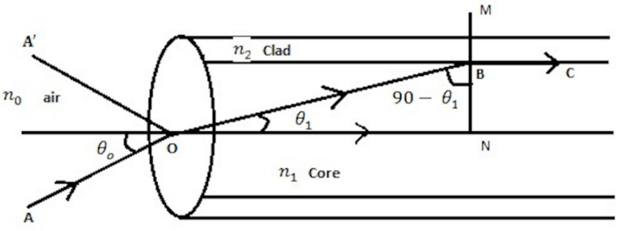


Figure 6.3: Ray propagation in the fiber

Consider an optical fiber with core made of refractive index n_1 & cladding made of material refractive index n_2. Let n_0 be the refractive index of the surrounding medium. Let a ray of light AO entering into core at an angle of incidence θ_0 w.r.t fiber axis. Then it is refracted along OB at an angle θ_1 & meet core-cladding interface at critical angle of incidence ($\theta_c = 90 - \theta_1$). Then the refracted ray grazes along BC. On applying Snell's law at O, we get

$$n_0 \, sin\theta_0 = n_1 \, sin\theta_1$$

$$\therefore \; sin\theta_0 = \frac{n_1}{n_0} \, sin\theta_1 \tag{6.2}$$

On applying Snell's law at point B, we get

$$n_1 \, sin(90^0 - \theta_1) = n_2 \, sin90^0$$

$$n_1 \, cos\theta_1 = n_2$$

$$\therefore \; cos\theta_1 = \frac{n_2}{n_1} \tag{6.3}$$

From trigonometric identity

$$sin^2\theta_1 + cos^2\theta_1 = 1$$

$$sin\theta_1 = \sqrt{1 - cos^2\theta_1}$$

using equation 1.25

$$sin\theta_1 = \sqrt{1 - \left(\frac{n_2}{n_1}\right)^2}$$

$$sin\theta_1 = \sqrt{\frac{{n_1}^2 - {n_2}^2}{{n_1}^2}}$$

$$sin\theta_1 = \frac{1}{n_1} \, \sqrt{{n_1}^2 - {n_2}^2} \tag{6.4}$$

use equation (1.26) in equation (1.24) we have,

$$sin\theta_0 = \frac{n_1}{n_0} \, \frac{1}{n_1} \, \sqrt{{n_1}^2 - {n_2}^2}$$

$$sin\theta_0 = \frac{1}{n_0} \, \sqrt{{n_1}^2 - {n_2}^2} \tag{6.5}$$

Numerical aperture $N.A = sin\theta_0$

$$N.A = \frac{1}{n_0} \, \sqrt{{n_1}^2 - {n_2}^2} \tag{6.6}$$

If the fiber is in air $n_0 = 1$ then,

$$N.A = sin\theta_0 = \sqrt{{n_1}^2 - {n_2}^2} \tag{6.7}$$

Light is transmitted through the fiber only when

$$\theta_i \; \leq \; \theta_0 \tag{6.8}$$

$$sin\theta_i \; \leq \; sin\theta_0 \tag{6.9}$$

$$sin\theta_i \; \leq \; \sqrt{{n_1}^2 - {n_2}^2} \tag{6.10}$$

$$sin\theta_i \; \leq \; N.A \tag{6.11}$$

This is the condition for propagation. Light will be transmitted through the optical fiber with multiple total internal reflections when the above condition is satisfied.

6.3.2 Fractional RI Change

Fractional index change (Δ) is defined as the ratio of difference in refractive indices of core & cladding to the refractive index of the core.

$$\Delta = \frac{n_1 - n_2}{n_1} \tag{6.12}$$

$$n_1 \, \Delta = n_1 - n_2 \tag{6.13}$$

6.3.3 Relation between NA and Δ

consider the equation

$$N.A = sin\theta_0 = \sqrt{{n_1}^2 - {n_2}^2}$$

$$N.A = \sqrt{(n_1 + n_2) \; (n_1 - n_2)} \tag{6.14}$$

For small difference of n_1 & n_2, we can have,

$$n_1 \approx n_2 \tag{6.15}$$

$$\therefore \; n_1 + n_2 \approx 2n_1$$

$$\therefore \; N.A = \sqrt{(2\,n_1) \; (n_1\,\Delta)}$$

$$N.A = \sqrt{2\,{n_1}^2\,\Delta}$$

$$N.A = n_1 \, \sqrt{2\,\Delta} \tag{6.16}$$

Thus the Numerical aperture can be increased by increasing the fractional index change.

6.4 Modes of propagation

Though optical fiber should support any numbers of rays for propagation practically. But it is found that the optical fiber allows only a certain restricted number of rays for propagation. The maximum number of rays or paths supported by the fiber for the propagation of light is called *Modes of propagation*.

V-number (Normalised Frequency of the fiber) An Optical fiber may be characterized by one more parameter called V-number. This determines the Number of modes supported by an optical fiber for the propagation.

$$V = \frac{\pi\, d}{\lambda}\, \sqrt{{n_1}^2 - {n_2}^2} \tag{6.17}$$

$$V = \frac{\pi\, d}{\lambda}\, N.A \tag{6.18}$$

here d is the diameter of the core, λ is wavelength, n_1 is the refractive index of the core and n_2 is the refractive index of the cladding. N.A is numerical Aperture. If the fiber is surrounded by a medium of refractive index n_0, then

$$V = \frac{\pi\, d}{\lambda}\, \frac{\sqrt{{n_1}^2 - {n_2}^2}}{n_0} \tag{6.19}$$

If $V \gg 1$, the number of modes supported by fiber can be determined using the formula

$$N \cong \frac{V^2}{2}$$

$$N \cong \frac{\pi^2\, d^2}{2\,\lambda^2} \left(\frac{{n_1}^2 - {n_2}^2}{n_0^2} \right) \tag{6.20}$$

6.4.1 Types of optical fibers

In any optical fiber, the whole material of the cladding has a uniform refractive index value. But the refractive index of the core material may either remain constant or subjected to variation in a particular pattern. The curve which represents the variation of refractive index with respect the radial distance from the axis of the fiber is called the *refractive index profile.* The optical fibers are classified under 3 categories,

1. Step index single mode fiber
2. Step index multi-mode fiber
3. Graded index multimode fiber

This classification is done depending on the refractive index profile, and the number of modes that the fiber can guide.

Step index single mode fiber

A single mode step index fiber consists of a very fine thin core of uniform RI surrounded by Cladding of RI lower than that of Core. Since there is abrupt change in the RI of Core and Cladding at the interface it is called step index fiber. Since the Core size is small the Numerical aperture is also small and hence support single mode. They accept light from LASER source. Splicing is difficult. They are used in submarine cables.

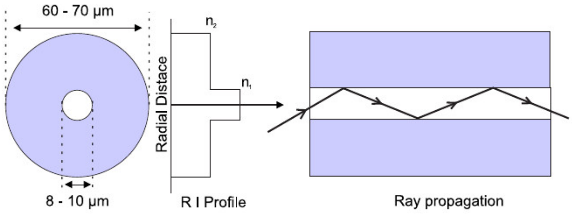


Figure 6.4: Step index single mode fiber

Step index multimode fiber

This is similar to single mode step index fiber with the exception that it has a larger core diameter. The core diameter is very large as compared to single mode optical fiber. A typical multimode step index fiber is as shown in figure. The numerical aperture is large because of large core size and thus support multimodes. They accept light from both LASER as well as from LED. They are used in data links.

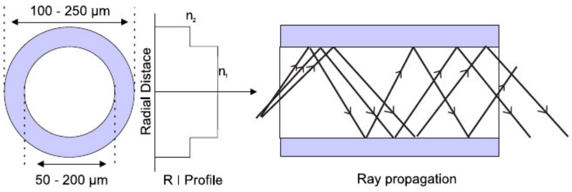


Figure 6.5: Step index multimode fiber

Graded index multimode fiber

A multimode fiber has concentric layers of RI is called GRIN fiber. It means the R I of the Core varies with distance from the fiber axis. The RI is maximum at the center and decreases with radial distance towards to core-cladding interface. The R I profile is as shown in fig. In GRIN fibers the acceptance angle and numerical aperture diminish with radial distance. The light transmission is as shown above. They accept light from both LASER as well as from LED. They are used for medium distance communication for example telephone link between central offices.

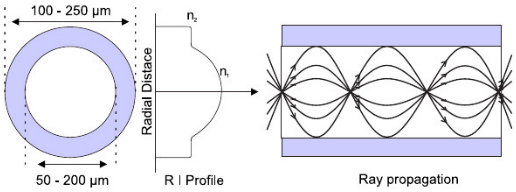


Figure 6.6: Graded index multimode fiber

6.5 Attenuation

The optical energy (signal) passing through the optical fiber gets reduced progressively. This is due to attenuation. It is also called the fiber loss or significant loss. The attenuation is measured in terms of attenuation co-efficient. The attenuation co-efficient α is defined as the ratio of optical power output to the optical power input for a fiber of length L and for a given wavelength of propagating light. It is expressed in dB/km. Attenuation co-efficient is given by

$$\alpha = \frac{-10}{L} \log_{10}\left(\frac{P_{out}}{P_{in}}\right) \quad dB/km \tag{6.21}$$

Here L is the length of the cable in km, P_{in} is Power of optical signal at launching end (input power) & P_{out} is Power of optical signal at receiving end (output power)

The attenuation in fibers gives is due to the following three losses

1. Absorption losses
2. Scattering loss (due to Rayleigh Scattering)
3. Geometric Effects (Radiation losses)

6.5.1 Absorption loss

In this type of loss, the loss of signal power occurs due to absorption of photons associated with the signal. Photons are absorbed either by impurities in the glass fiber or by pure glass material itself. Absorption loss is wavelength dependent.Thus absorption loss is classified in to two types.

Extrinsic absorption : Extrinsic loss in an optical fiber is due to the absorption of light by the impurities such as hydroxide ions and transition metal ions such as iron, chromium, cobalt and copper.

Intrinsic absorption Intrinsic loss in fiber is due to the absorption of light by the material of the fiber glass itself. The intrinsic losses are insignificant.

6.5.2 Scattering loss

Light traveling through the core can get scattered by impurities or small regions with sudden change in refractive index. Rayleigh scattering varies as $\alpha = \frac{1}{\lambda^4}$ and leads to significant power loss at smaller wavelengths. The scattering results in loss of photons. Rayleigh scattering is responsible for maximum losses in optical fibers.

6.5.3 Geometric effects

These may occur due to manufacturing defects like irregularities in fibre dimensions during drawing process or during coating, cabling or insulation processes. The microscopic bends are the bends with radii greater than fiber diameter. The microscopic bends couple light between the various guided modes of the fiber and some of them then leak through the fiber.

6.6 Applications of Optical Fibers

6.6.1 Point to point communication using Optical Fibers

In an optical fiber communication system, the input signals (audio, video or other digital data) are used to modulate light from a source like a LED or a semiconductor LASER and is transmitted through optical fiber. At the receiving end the signal is demodulated to reproduce the input signal. If data transfer takes place between only two devices then, it is called point to point communication.

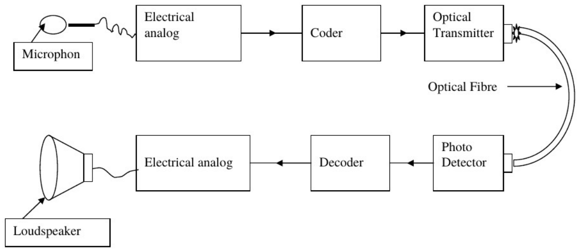


Figure 6.7: Point to point fiber optic communication system

Optical fiber communication process : The communication using Optical fiber is as follows. First voice is converted into electrical signal using a transducer. It is digitized using a Coder. The digitized signal, which carries the voice information, is fed to an optical transmitter. The light source in optical transmitter (LED or LASER Diode) emits modulated light, which is transmitted through the optical fiber. At the other end the modulated light signal is detected by a photo detector and is decoded using a decoder. Finally the information is converted into analog electrical signal and is fed to a loud speaker, which converts the signal to voice (sound).

Advantages

1. Optical fibers can carry very large amounts of information in either digital or analog form.
2. The raw material for optical fiber is of low cost and abundant.
3. It has low cost /meter/ channel
4. Cables are very compact
5. Signals are protected from radiation from lightning or sparking
6. There is no energy radiation from fiber
7. No sparks are generated

Disadvantages

1. The optical connectors are very costly
2. Maintenance cost is high
3. They cannot be bent too sharply
4. They under go structural changes with temperature

6.6.2 Fiber Optic Displacement Sensor

Principle: Light is sent through a transmitting fiber and is made to fall on a moving target. The reflected light from the target is sensed by a detector. With respect to intensity of light reflected from its displacement of the target is measured.

Construction: It consists of a bundle of transmitting fibers coupled to the laser source and a bundle of receiving fibers coupled to the detector as shown in the figure. The axis of the transmitting fiber and the receiving fiber with respect to the moving target can be adjusted to increase the sensitivity of the sensor.

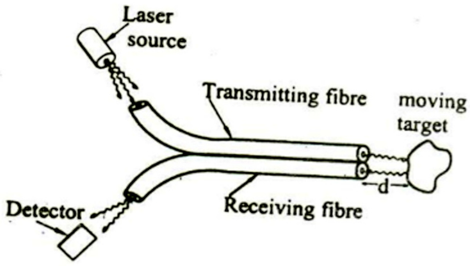


Figure 6.8: Fiber Optic Displacement Sensor

Working: Light from the source is transmitted through the transmitting fiber and is made to fall on the moving target. The light reflected from the target is made to pas through the receiving fiber and the same is detected by the detector.Based on the intensity of the light received, the displacement of the target can be measured, (i.e.) if the received intensity is more than we can say that the target is moving towards the sensor and if the intensity is less, we can say that the target is moving away from the sensor.

6.6.3 Merits and Demerits of Fiber Optic Sensors

Merits

1. It is immune from nearby EM (electromagnetic) and stray radiation.
2. It can be used in environments where high levels of electrical interference exists or where intrinsic safety is a concern.
3. It is light in weight and compact in size.
4. It is cheaper due to low manufacturing cost.
5. It offers high sensitivity and hence very small changes can also be measured.
6. It offers wide dynamic range and large bandwidth.
7. It offers multiplexing and remote sensing capabilities
8. It is tolerent against high temperature (i.e. >1450 deg centigrade) and corrosive environments.
9. It is safe and suitable to be used in extreme vibration and harse environments.

Demerits

1. It is very expensive.
2. Detection systems may be complex.
3. The users require training before they start using such sensor types.
4. Some fiber optic temperature sensors are expensive.
5. It requires precise installation methods or procedures.

6.7 Model Questions

1. Define the terms: (i) angle of acceptance, (ii) numerical aperture, (iii) fractional index change (iv) modes of propagation & (v) refractive index profile.
2. Obtain an expression for numerical aperture and arrive at the condition for propagation.

3. Give the relation between Numerical aperture and Fractional index change.

4. What is refractive index profile? Discuss different types of optical fibers with suitable diagrams.

5. What is attenuation? Explain the factors contributing to the fiber loss.

6. What is attenuation coefficient? Mention the expression for the attenuation coefficient.

7. What are the advantages of optical communications over other conventional types of communication?

8. Discuss point to point optical fiber communication system and mention its advantages over the conventional communication system.

9. Discuss the advantages and disadvantages of an optical communication.

6.8 Numerical Problems

1. Calculate the numerical aperture and angle of acceptance for an optical fiber having refractive indices 1.563 and 1.498 for core and cladding respectively.

2. The refractive indices of the core and cladding of a step index optical fiber are 1.45 and 1.4 respectively and its core diameter is $45\mu m$. Calculate its fractional refractive index change and numerical aperture.

3. Calculate numerical aperture, acceptance angle and critical angle of a fiber having a core RI 1.50 and cladding RI 1.45.

4. An optical fiber has a numerical aperture of 0.32. The refractive index of cladding is 1.48. Calculate the refractive index of the core, the acceptance angle of the fiber and the fractional index change.

5. An optical signal propagating in a fiber retains 85% of input power after traveling a distance of 500 m in the fiber. Calculate the attenuation coefficient.

6. An optical fiber has core RI 1.5 and RI of cladding is 3% less than the core index. Calculate the numerical aperture, angle of acceptance critical angle.

7. The numerical aperture of an optical fiber is 0.2 when surround by air. Determine the RI of its core, given the RI of the cladding is 1.59. Also find the acceptance angle when the fiber is in water of RI 1.33.

8. The angle of acceptance of an optical fiber is 30^0 when kept in air. Find the angle of acceptance when it is in medium of refractive index 1.33.

9. Calculate NA, V-number and number of modes in an optical fiber of core diameter $50\mu m$, core and cladding refractive indices 1.41 and 1.4 respectively at wavelength 820 nm.

10. For a step index optical fiber RI of core is 1.45 and RI of cladding is 1.40 and its core diameter is $45\mu m$. Calculate its relative refractive index difference, V-number at wavelength 1000 nm and the number of modes.

11. Calculate the number of modes of an optical fiber will transmit using the following data $n_{core} = 1.50$, $n_{clad} = 1.48$, core radius = $50\mu m$, $\lambda = 1\mu m$.

12. An optical fiber of 600 m long has input power of 120 mW which emerges out with power of 90 mW. Find attenuation in fiber.

13. The attenuation of light in an optical fiber is 3.6 dB/km. What fraction of its initial intensity is remains after i) 1 km and ii) 3 km ?

14. The attenuation of light in an optical fiber is 2.2 dB/km. What fraction of its initial intensity is remains after i) 2 km and ii) 6 km ?

Part IV

Module - 4 - Maxwell’s equations and EM Waves

Chapter 7

Maxwell's Equations

7.1 Fundamentals of vector calculus

7.1.1 Dot product or Scalar product

The dot product of two vectors is defined as follows

$$\vec{a}.\vec{b} = abcos\theta \tag{7.1}$$

here θ is the angle between two vectors. a and b are the magnitudes of $\vec{a}$ and $\vec{b}$. If $\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{z}$ and $\vec{b} = b_x\hat{\imath} + b_y\hat{\jmath} + b_z\hat{z}$ then the dot product or scalar product is given by

$$\vec{a}.\vec{b} = a_xb_x + a_yb_y + a_zb_z \tag{7.2}$$

The dot product of two vectors is a scalar quantity.

Physical Significance The dot product is mathematically put forward and could be applied in physics under suitable circumstances. For example the work done is maximum when the displacement is along the force. Thus work done is defined as the dot product of force ($\vec{F}$) and displacement($\vec{d}$) and is a scalar quantity. Hence $W = \vec{F}.\vec{d}$.

7.1.2 Vector product or Cross product

The vector product of two vectors is defined as follows

$$\vec{a} \times \vec{b} = a\ b\ sin\theta\ \hat{n} \tag{7.3}$$

here θ is the angle between two vectors. a and b are the magnitudes of $\vec{a}$ and $\vec{b}$. $\hat{n}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$. If $\vec{a} = a_x\hat{\imath}+a_y\hat{\jmath}+a_z\hat{z}$ and $\vec{b} = b_x\hat{\imath}+b_y\hat{\jmath}+b_z\hat{z}$ then their cross product is given by

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} \tag{7.4}$$

The cross product of two vectors is a vector quantity.

Physical Significance The cross product is put forward in mathematics an could be applied in physics under suitable circumstances. For a rotating body the moment of linear momentum is the angular momentum. The angular momentum acts in a direction perpendicular to momentum and the radius vector. Thus angular momentum ($\vec{L}$) is given by the cross product of radius vector ($\vec{r}$) and linear momentum ($\vec{p}$) and hence $\vec{L} = \vec{r} \times \vec{p}$.

7.1.3 Scalar field

It is a function of a space whose value at each point is a scalar quantity. For example potential setup by a charge in space.

7.1.4 Vector field

It is a function of a space whose value at each point is a vector quantity. Consider a region in the flowing water. Each and every point can be associated with a vector whose magnitude represents the speed of flow and direction gives the direction of flow. Thus the whole region could be imagined filled with vectors and is an example of vector field. Consider a region surrounding a point charge. The electric field at each and every point surrounding the charge could be represented by vectors and hence is a vector field.

7.1.5 The ∇ Operator

In mathematics the following operator is used called ∇ operator. When this operator acts on a scalar quantity it instructs to differentiate the scalar quantity. The operation of ∇ on a scalar quantity results in a vector quantity. The ∇ operator is given by

$$\nabla = \hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z} \tag{7.5}$$

Let T be a scalar function. Then ∇T states that the ∇ acts on T. There are three ways in which ∇ can act.

1. On a scalar function ∇T called the **Gradient**.
2. On a vector function via the dot product $\nabla.\vec{A}$ called the **Divergence**.
3. On a vector function via the cross product $\nabla\times\vec{A}$ called the **Curl**.

7.1.6 The Gradient

Consider a scalar function V. The operator ∇ acting on the scalar function V is given by

$$\nabla V = \hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z} \tag{7.6}$$

The gradient ∇V points along the maximum variation of the function V and the magnitude of ∇V gives the rate of change in the maximal direction.

Physical significance: Let us consider a positive point charge in space. Let the potential set up by the charge in the surrounding be V and is a scalar quantity. The potential decreases as the distance from the charge increases. Thus the gradient of potential results in the electric field strength which is a vector quantity. This could be written as

$$\vec{E} = -\frac{\partial V}{\partial r}\hat{r} \tag{7.7}$$

Here r is the position vector and $\hat{r}$ is the unit vector along position vector. The negative sign indicates the decrease in potential. Thus the above equation could be written as

$$\vec{E} = -\nabla V = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right] \tag{7.8}$$

Thus the Electric field strength is defined as negative of gradient of potential also known as $grad\ V$.

7.1.7 The Divergence

The divergence of a vector field is mathematically written as $\nabla.\vec{E}$. The vector field E is represented by $\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$. From the definition of the ∇ we can construct divergence as

$$\nabla.\vec{E} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right).\left(E_x\hat{i} + E_y\hat{j} + E_z\hat{k}\right)$$

$$\nabla.\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \tag{7.9}$$

From the equation 7.9 we observe that the divergence of a vector field is a scalar quantity.

Physical significance : The physical significance of the divergence of a vector function is it measures how much the vector E spreads out (diverges) from a point of consideration. For example if we consider a positive charge in space the field lines diverge and hence it is **positive divergence**. For a negative charge the field lines converge and hence it is **negative divergence**. If the field lines or parallel then it is**zero divergence**. See fig. 7.1.

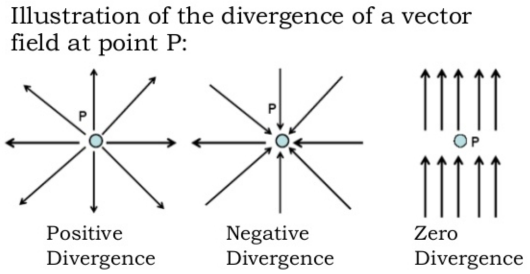


Figure 7.1: Positive, Negative and Zero Divergence

7.1.8 The Curl

The curl of a vector field is could be constructed as follows

$$\nabla \times \vec{H} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \times \left(H_x\hat{i} + H_y\hat{j} + H_z\hat{k}\right)$$

$$\nabla \times \vec{H} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} \tag{7.10}$$

The equation 7.10 represents curl of $\vec{H}$ and also it is evident that curl of a vector is a vector quantity.

Physical significance : The curl of a vector function is a measure how much field swirls (curls) around the point of consideration. Consider a wire carrying electric current. This sets magnetic field surrounding the wire. Consider a point on the wire. The magnetic field lines curl or swirl around the point. Higher the value of $\vec{H}$ around the point stronger will be the curl. If the field lines purely parallel then it represents zero curl around the point. See fig 7.2.

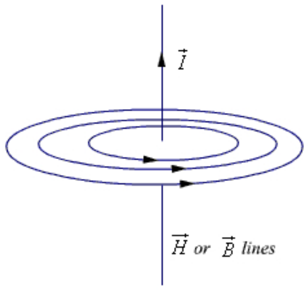


Figure 7.2: Curl of a magnetic field

7.2 Line, Surface and Volume integrals

7.2.1 Line integral

Line integral is an expression of the form

$$\int_{P}^{Q} \vec{A}.\vec{dl} \tag{7.11}$$

here $\vec{A}$ represents the vector field and $\vec{dl}$ represents a infinitesimally small length at a point M along the path PQ in the field. The dot product of $\vec{A}$ and $\vec{dl}$ is given by $\vec{A}.\vec{dl} = A\ dl\ Cos\theta$, Here θ is the angle made $\vec{dl}$ with $\vec{A}$. For a closed path the integral is written as

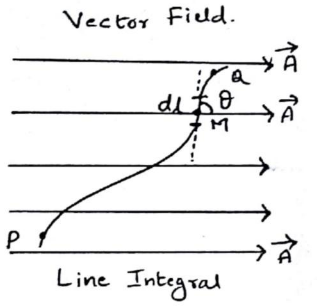


Figure 7.3: Line Integral

$$\oint \vec{A}.\vec{dl} \tag{7.12}$$

$\oint$ is the symbol used for closed contour integral. This is also called as circulation of $\vec{A}$ around the closed path. The line integral concept can be applied to calculate the potential difference between to points in an electric field.

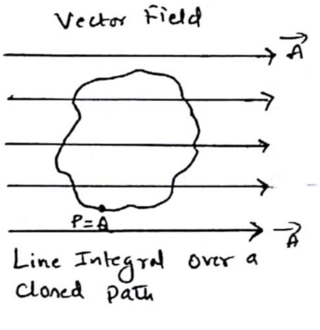


Figure 7.4: Circulation of vector

7.2.2 Surface integral

Consider a surface of area S in a vector field $\vec{A}$. consider a small infinitesimal area dS on the surface around point M as in the figure. Consider $\hat{n}$ a unit vector normal to dS and $dS\ \hat{n}$ represents area vector $\vec{dS}$. The surface integral over the entire surface S is given by

$$\int_{s} \vec{A}.\vec{dS} \tag{7.13}$$

Here $\int_s$ is the symbol used for surface integral. The surface integral gives the net outward flux of the vector field through the surface. For a closed surface the surface integral is given by

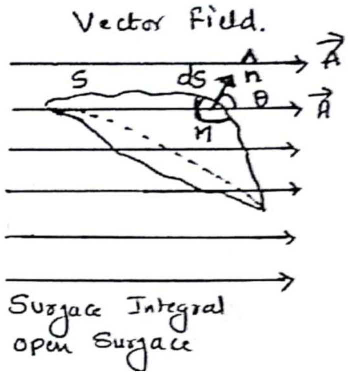


Figure 7.5: Surface Integral

$$\oint_{s} \vec{A}.\vec{dS} \tag{7.14}$$

In case of surface integral for a closed surface the $\hat{n}$ chosen outwards. The surface integral could be applied to calculate the net flux of the electric field through a surface in the electric field.

7.2.3 Volume integral

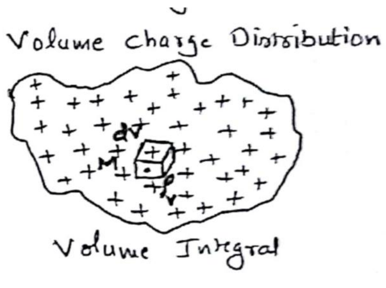


Figure 7.6: Volume Integral

Consider a volume charge distribution in which charges are continuously distributed. Let v be the volume through which the charges are distributed. Consider a point M inside the charge distribution. Let dv be a small volume around a point M. let ρ_v be the density of charges at M

and is a scalar quantity. The net charge in the volume is given by volume integral of the form

$$\oint_v \rho_v dv \tag{7.15}$$

here $\oint_v$ is the symbol for volume integral.

7.3 Some Theorems of Electrostatics, Electricity, Magnetism and Electromagnetic induction

7.3.1 Gauss flux theorem - Gauss' law in electrostatics

Consider a region in space consisting of charges. Let a surface of any shape enclose these charges and is called a Gaussian surface. Let Q be the net charge enclosed by the Gaussian surface S. The closed surface could be considered to be made up of number of elementary surfaces dS. If $\vec{D}$ is the electric flux density at dS then the surface integral gives the total electric flux over the surface S could be obtained as

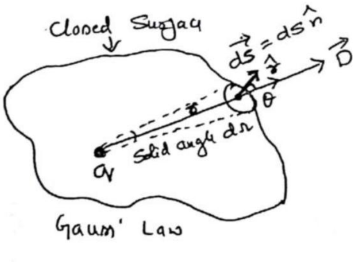


Figure 7.7: Gauss' Flux Theorem - Electrostatics

$$\phi = \oint_s \vec{D}.\vec{dS} = \sum q = Q \tag{7.16}$$

here ϕ is the total flux and $\sum q = (q_1 + q_2 + ...)$ is the total charge enclosed by the surface.

7.3.2 Gauss Divergence Theorem

Divergence of $\vec{D}$

Consider a vector field $\vec{D}$. Consider a point P in the vector field. Let ρ_v be the density of charges at the point P. It can be shown that the divergence of the $\vec{D}$ is given by

$$\nabla.\vec{D} = \rho_v \tag{7.17}$$

This is also the Maxwell's first equation.

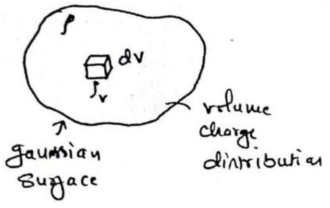


Figure 7.8: Gauss divergence theorem

Statement: The Gauss divergence theorem states that the integral of the normal component of the flux density over a closed surface of any shape in an electric field is equal to the volume integral of the divergence of the flux throughout the space enclosed by the Gaussian surface. Mathematically

$$\oint_s \vec{D}.\vec{dS} = \oint_v (\nabla.\vec{D})\ dv \tag{7.18}$$

Proof

Consider a volume v enclosed by a Gaussian surface S as show in the fig. 7.8. Let a charge dQ be enclosed by a small volume dv inside the Gaussian surface. If ρ is the density of charges and may vary inside the volume v then the charge density associated with volume dv is given by

$$\rho_v = \frac{dQ}{dv}$$

Thus

$$dQ = \rho_v\ dv$$

Thus the total charge enclosed by the Gaussian surface is give by

$$Q = \oint_v dQ = \oint_v \rho_v\ dv$$

Substituting for ρ_v from Maxwell's First equation 7.18 we get

$$Q = \oint_v (\nabla.\vec{D})\ dv$$

According to Gauss' law of electrostatics we have

$$Q = \oint_s \vec{D}.\vec{dS}$$

Thus equating the equations for Q we get

$$\oint_s \vec{D}.\vec{dS} = \oint_v (\nabla.\vec{D})\ dv \tag{7.19}$$

Thus Gauss divergence theorem. Divergence theorem relates the surface integral with volume integral.

7.3.3 Stokes' Theorem

Stokes, theorem relates surface integral with line integral (Circulation of a vector field around a closed path).

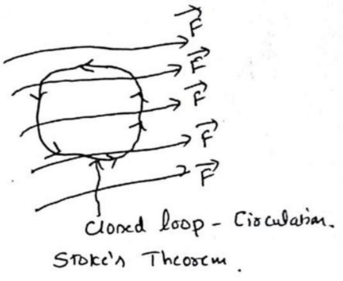


Figure 7.9: Stokes' theorem

Statement: The surface integral of curl of $\vec{F}$ throughout a chosen surface is equal to the circulation of the $\vec{F}$ around the boundary of the chosen surface.

Mathematically

$$\int_s (\nabla \times \vec{F}).\vec{dS} = \oint \vec{F}.\vec{dl} \tag{7.20}$$

7.3.4 Gauss' law of Magnetostatics

Consider a closed Gaussian surface of any shape in a magnetic field. The magnetic fields lines exist in closed loops. Hence for every flux line that enters the closed surface a flux line emerges out else where. Thus for a closed surface in a magnetic field the total inward flux(Positive) is equal to total outward flux(Negative). Thus the net flux through the Gaussian surface is zero. Thus it could be written

$$\oint_s \vec{B}.\vec{dS} = 0 \tag{7.21}$$

Here $\vec{B}$ magnetic flux density. Applying Gauss divergence theorem we get

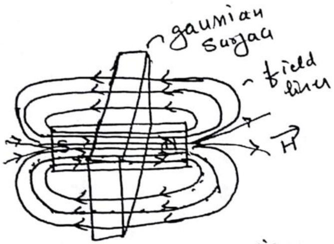


Figure 7.10: Gauss' Flux Theorem - Magnetostatics

$$\oint_s \vec{B}.\vec{dS} = \oint_v (\nabla.\vec{B})\ dv = 0$$

Hence it could be written

$$\nabla.\vec{B} = 0 \tag{7.22}$$

This is one of the Maxwell's equations.

7.3.5 Amperes Law

Statement: The circulation of magnetic field strength $\vec{H}$ along a closed path is equal to the net current enclosed (I_{enc}) by the loop. Mathematically

$$\oint \vec{H}.\vec{dl} = I_{enc} \tag{7.23}$$

By applying stokes' theorem we get

$$\int_s (\nabla \times \vec{H}).\vec{dS} = I_{enc} \tag{7.24}$$

The equation for I_{enc} could be obtained as

$$I_{enc} = \oint_s \vec{J}.\vec{dS} \tag{7.25}$$

Equating equations 7.24 and 7.25 we get

$$\int_s (\nabla \times \vec{H}).\vec{dS} = \oint_s \vec{J}.\vec{dS}$$

Thus we get the amperes law as

$$\nabla \times \vec{H} = \vec{J} \tag{7.26}$$

Thus Amperes circuital law and another Maxwell's equation.

7.3.6 Biot-Savart Law

Consider a portion of a conductor carrying current I. Let dl be infinitesimally small elemental length of the conductor at M. Consider a point P near The conductor. Let $\vec{MP}$ be the the vector joining the element with the point and of length r with $\hat{r}$ being the unit vector. θ is the angle made by MP with the element. Biot-Savart law states the magnitude and direction of the small magnetic field at P due to the elemental length dl of the current carrying conductor.

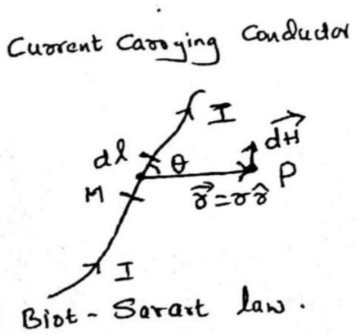


Figure 7.11: Biot-Savart Law

The magnitude of the magnetic field $\vec{dH}$ is

1. Proporitonal to the length of the element dl

2. Proportional to the current through the element I.
3. Proportional to the Sine of the angle θ, $Sin(\theta)$.
4. Inversely proportional to the square of the distance r.

The direction of the magnetic field $\vec{dH}$ is perpendicular to the plane containing both the element and the vector $\vec{r}$. Mathematically we get

$$dH \propto \frac{I\ dl\ Sin(\theta)}{r^2}$$

$$dH = \frac{\mu_0}{4\pi}\frac{I\ dl\ Sin(\theta)}{r^2} \tag{7.27}$$

Here $\frac{1}{4\pi}$ is the proportionality constant. the above equation could be expressed in the vector form as

$$\vec{dH} = \frac{\mu_0}{4\pi}\frac{I\ \vec{dl} \times \hat{r}}{r^2} \tag{7.28}$$

Thus the Biot-Savart Law. It could also be written as

$$\vec{dH} = \frac{\mu_0}{4\pi}\frac{I\ \vec{dl} \times \vec{r}}{r^3} \tag{7.29}$$

7.3.7 Faraday's Laws of electro-magnetic induction

Statement

1. When ever there is a change in magnetic flux linked with the circuit an **emf** (e) is induced and is equal to rate of change of magnetic flux.
2. The emf induced is in such a direction that it apposes the cause.

Mathematically the induced emf is given by

$$e = -\frac{d\phi}{dt} \tag{7.30}$$

Here ϕ is magnetic flux linked with the circuit. For a coil of N turns the induced emf due to rate of change of flux is given by

$$e = -N\frac{d\phi}{dt} \tag{7.31}$$

Faraday's law in integral and differential forms

For a conducting loop linked with changing magnetic flux, the rate of change flux is

$$\frac{d\phi}{dt} = \int_s \frac{\partial \vec{B}}{\partial t}.\vec{dS} \tag{7.32}$$

The induced emf in the the circuit is given by

$$e = \oint \vec{E}.\vec{dL} \tag{7.33}$$

Substituting the above in the equation 7.30 we get

$$\oint \vec{E}.\vec{dL} = -\int_s \frac{\partial \vec{B}}{\partial t}.\vec{dS} \tag{7.34}$$

Using the Stokes' theorem

$$\oint \vec{E}.\vec{dL} = \int_s (\nabla \times \vec{E}).\vec{dS} \tag{7.35}$$

and hence we can write

$$\int_s (\nabla \times \vec{E}).\vec{dS} = -\int_s \frac{\partial \vec{B}}{\partial t}.\vec{dS} \tag{7.36}$$

Thus finally it reduces to

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{7.37}$$

Thus Faraday's law in differential (Point form) and one of the Maxwell's equations.

7.4 Equation of continuity

In all processes involving motion of charge carriers the net charge is always conserved and is called the law of conservation of charges.

Let us consider a volume V. Let the charges flow into and out of the volume V. Then the equation for the law of conservation could be written in the integral form as

$$\oint_s \vec{J}.\vec{dS} = -\frac{\partial}{\partial t}\int_v \rho_v\ dV \tag{7.38}$$

ρ_v is the volume density of charges and $\vec{J} = Ne\vec{v} = \rho_v\vec{v}$ is the current density.The negative sign indicates that the current density is due to the decrease in positive charge density inside the volume. Using the Gauss divergence theorem we can write

$$\oint_s \vec{J}.\vec{dS} = \oint_v (\nabla.\vec{J}).dV$$

Thus the equation 7.38 could be written as

$$\oint_v (\nabla.\vec{J}).dV = -\frac{\partial}{\partial t}\int_v \rho_v\ dV$$

The above equation could be reduced to

$$\oint_v (\nabla.\vec{J}).dV = -\int_v \frac{\partial \rho_v}{\partial t}\ dV$$

Thus the equation of continuity could be written as

$$\nabla.\vec{J} = -\frac{\partial \rho_v}{\partial t} \tag{7.39}$$

Equation 7.39 represents the law of conservation of charges.

Discussion on equation of continuity :

In case of DC circuits for steady currents the inward flow of charges is equal to the outward flow through a closed surface and hence $\frac{\partial \rho_v}{\partial t} = 0$. Thus the equation of continuity becomes $\nabla.\vec{J} = 0$.

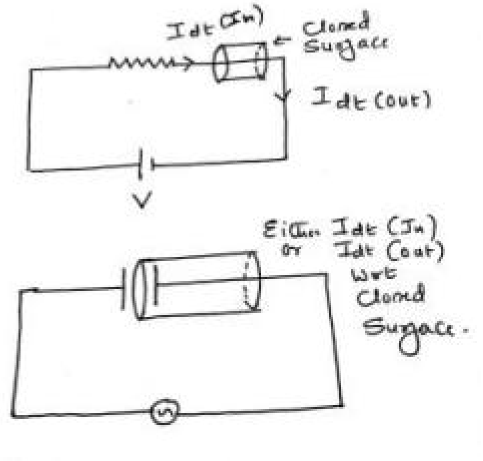


Figure 7.12: DC and AC circuits - Continuity equation

In case of AC circuits containing capacitors the equation $\nabla.\vec{J} = 0$ fails as follows. During the positive half cycle, say, the capacitor charges. If we imagine a closed surface enclosing the capacitor plate and the attached conductor there will be inward flow of charges into the closed surface but no outward flow. Thus in order to rescue the equation of continuity Maxwell introduced the concept of displacement current density.

7.5 Displacement Current

7.5.1 Definition

Displacement current density is a correction factor introduced by Maxwell in order to explain the continuity of electric current in time-varying circuits. It has the same unit as electric current density. Displacement current is associated with magnetic field but it does not describe the flow of charge.

7.5.2 Maxwell-Ampere Law

Introducing the concept of displacement current for time varying circuits, Maxwell suggested corrections to the Amperes law. According to Gauss' Law

$$\nabla.\vec{D} = \rho_v$$

Differentiating the above equation with respect to time

$$\frac{\partial}{\partial t}\left(\nabla.\vec{D}\right) = \frac{\partial \rho_v}{\partial t}$$

$$\nabla.\frac{\partial \vec{D}}{\partial t} = \frac{\partial \rho_v}{\partial t} \tag{7.40}$$

The equation of continuity is given by

$$\nabla.\vec{J} = -\frac{\partial \rho_v}{\partial t}$$

Hence equation 7.40 could be written as

$$\nabla.\vec{J} = -\nabla.\left(\frac{\partial \vec{D}}{\partial t}\right)$$

$$\nabla.\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) = 0$$

Hence for time varying circuits $\nabla.\vec{J} = 0$ does not hold good and instead $\nabla.\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) = 0$ has to be used. Also $\vec{J}$ in Amperes Circuital law $\nabla \times \vec{H} = \vec{J}$ has to be replace with $\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$ Thus the Maxwell-Ampere law is given by

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \tag{7.41}$$

In the above equation $\frac{\partial \vec{D}}{\partial t}$ is called displacement current.

7.5.3 Expression for Displacement current

Consider an AC circuit containing a capacitor as shown in the figure 7.13

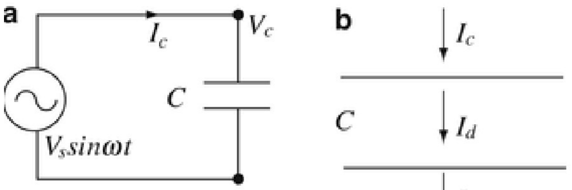


Figure 7.13: Displacement current

The displacement current in terms of displacement current density is given by

$$I_D = \left(\frac{\partial \vec{D}}{\partial t}\right).A \tag{7.42}$$

Here A is the area of the capacitor plates.The elecric flux density D is give by

$$D = \epsilon E \tag{7.43}$$

Here E is the electric field strength which is given by

$$E = \frac{V}{d} \tag{7.44}$$

Here d is the separation between the capacitor plates. V the applied potential is given by

$$V = V_s \ e^{jwt} \tag{7.45}$$

Using equations 7.43, 7.44 and 7.45 we get

$$D = \frac{\epsilon}{d} V_s \ e^{j\omega t} \tag{7.46}$$

Substituting for D in equation 7.42 from equation 7.46, we get

$$I_D = \frac{\partial}{\partial t}\left(\frac{\epsilon}{d} \ V_s \ e^{j\omega t}\right).A$$

Executing differentiation the displacement current is given by

$$I_D = \frac{j\omega\epsilon A}{d} \ V_s \ e^{jwt} \tag{7.47}$$

7.6 Maxwell’s Equations

Using the laws and theorems discussed in this chapter Four Maxwell’s equations for time-varying fields could be written as

1. Gauss’ Law of Electrostatics $\nabla.\vec{D} = \rho_v$
2. Faraday’s Law $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
3. Gauss’ Law of Magnetic fields $\nabla.\vec{B} = 0$
4. Maxwell - Ampere Law $\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

The Four Maxwell’s equations for static fields could be written as

1. $\nabla.\vec{D} = \rho_v$
2. $\nabla \times \vec{E} = 0$
3. $\nabla.\vec{B} = 0$
4. $\nabla \times \vec{H} = \vec{J}$

The above equations are used to study the electromagnetic waves.

Chapter 8

Electromagnetic waves

8.1 Introduction

The existence of EM waves was predicted by Maxwell theoretically using the point form of Faraday's Law of electromagnetic induction. As per Faraday's law a time varying magnetic field induces electric field which varies with respect to space and time. The reverse is also evident from the equations. Thus Electromagnetic wave is the propagation of energy in terms of varying electric and magnetic fields which are in mutually perpendicular directions and perpendicular to the direction of propagation.

8.2 Wave equation for EM waves in vacuum in terms of electric field using Maxwell's Equations

Consider the Maxwell's equations

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{8.1}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \tag{8.2}$$

Substituting $D = \epsilon E$ and $B = \mu H$ in the above equations we get

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \tag{8.3}$$

$$\nabla \times \vec{H} = \vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t} \tag{8.4}$$

To derive wave equation in terms of electric field, the term $\vec{H}$ has to be eliminated. Taking curl on both sides the equation 8.3 we get

$$\nabla \times \nabla \times \vec{E} = -\mu \frac{\partial}{\partial t}\left(\nabla \times \vec{H}\right) \tag{8.5}$$

According to vector analysis $A \times (B \times C) = B(A.C) - C(A.B)$. Thus

$$\nabla \times \nabla \times \vec{E} = \nabla\left(\nabla.\vec{E}\right) - \nabla^2 \vec{E}$$

As per the Maxwells equation $\nabla.\vec{D} = \rho_v$. Since $D = \epsilon E$ it could be written as $\nabla.\vec{E} = \frac{\rho_v}{\epsilon}$. Substituting in the above equation we get

$$\nabla \times \nabla \times \vec{E} = \nabla\left(\frac{\rho_v}{\epsilon}\right) - \nabla^2 \vec{E} \tag{8.6}$$

Substituting equation 8.6 in equation 8.5 we get

$$\nabla\left(\frac{\rho_v}{\epsilon}\right) - \nabla^2 \vec{E} = -\mu \frac{\partial}{\partial t}\left(\nabla \times \vec{H}\right) \tag{8.7}$$

Substituting equation 8.4 in 8.7 we have

$$\nabla\left(\frac{\rho_v}{\epsilon}\right) - \nabla^2 \vec{E} = -\mu \frac{\partial}{\partial t}\left(\vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t}\right) \tag{8.8}$$

the above equation could be rewritten as

$$\nabla^2 \vec{E} - \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = \mu \frac{\partial \vec{j}}{\partial t} + \nabla\left(\frac{\rho_v}{\epsilon}\right) \tag{8.9}$$

The LHS in equation 8.9 represents a propagating wave and the RHS the source of origin of the wave. Here μ and ϵ are respectively Absolute permeability and Absolute permittivity of isotropic homogeneous medium. In case of propagation of EM wave in free space ($\vec{J} = 0, \rho_v = 0$) equation 8.9 reduces to

$$\nabla^2 \vec{E} - \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \tag{8.10}$$

Hence the electromagnetic wave equation in free space. Comparing the above equation with the general wave equation we get the velocity of the EM wave

$$\frac{1}{v^2} = \mu\epsilon \tag{8.11}$$

hence velocity of the EM wave

$$v = \frac{1}{\sqrt{\mu\epsilon}} \tag{8.12}$$

The velocity of propagation of EM Wave in vacuum

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \approx 3 \times 10^8 ms^{-1} \tag{8.13}$$

8.3 Plane electromagnetic waves in vacuum

Electromagnetic waves that travels in one direction and uniform in the other two orthogonal directions is called plane electromagnetic waves. For example consider a plane electromagnetic wave traveling along z axis the electric and magnetic vibrations are uniform and confined to x-y plane.

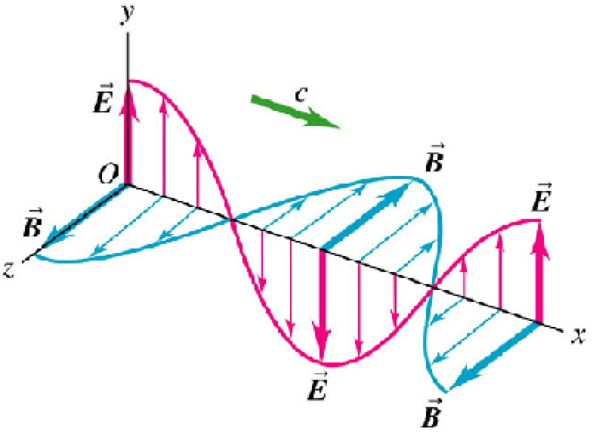


Figure 8.1: Plane Electromagnetic Waves

Consider a plane electromagnetic wave propagating along +ve x-axis. If the time varying electric and magnetic fields are along y and z axes respectively then we can write

$$\vec{E} = A\ Cos\left[\frac{2\pi}{\lambda}(x - ct)\right]\hat{i} \tag{8.14}$$

$$\vec{B} = \frac{1}{c}A\ Cos\left[\frac{2\pi}{\lambda}(x - ct)\right]\hat{j} \tag{8.15}$$

The ratio of the amplitudes of Electric and Magnetic fields from equations 8.14 and 8.15 is given by

$$\frac{E_y}{B_z} = c \tag{8.16}$$

Here 'c' is the velocity of light.

8.4 Polarization of Elecromagnetic waves

8.4.1 Transverse nature of electromagnetic waves

The electric and magnetic variations are mutually perpendicular and perpendicular to the direction of propagation. Thus electromagnetic waves are transverse in nature. Electromagnetic waves also exhibit polarization. Consider an electromagnetic wave propagating along z-axis. The the electric field vector of this electromagnetic wave makes an angle $theta$ with respect to x-axis, say. This electric vector could be resolved into two perpendicular components $\vec{E}_x$ and $\vec{E}_y$ along x and y axes respectively. Based on the magnitudes of the components and the phase difference between the components there are three kinds of polarization of electromagnetic waves. They are

1. Linearly Polarized EM waves
2. Circularly Polarized EM waves
3. Electrically Polarized EM waves

Linear polarization In case of linear polarization the amplitudes of $\vec{E}_x$ and $\vec{E}_y$ may or may not be equal and they are in phase(in unison). Thus the projection of the resultant $\vec{E}$ on a plane (x-y plane) perpendicular to the direction of propagation is a straight line. Thus linear polarization.

Circular polarization In case of circular polarization the amplitudes of $\vec{E}_x$ and $\vec{E}_y$ are equal in magnitude and the phase difference is 90°. Thus the projection of the resultant traces a circle on the plane perpendicular to the direction of propagation. Thus Circular polarization.

Elliptical polarization In case of circular polarization the amplitudes of $\vec{E}_x$ and $\vec{E}_y$ are unequal in magnitude and the phase difference is 90°. Thus the projection of the resultant traces an ellipse on the plane perpendicular to the direction of propagation. Thus Circular polarization.

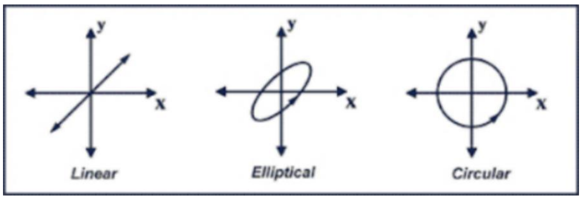


Figure 8.2: Polarization of Electromagnetic Waves

The linear, circular and elliptical polarization are as shown in the figure 8.2.

Part V

Module - 5 - Semiconductor and Devices

Chapter 9

Semiconductors

9.1 Introduction

Semiconductors are the materials which posses negative temperature co-efficient of resistance. Their electrical properties are different from that conductors and insulators. The pure form of semiconductors are known as intrinsic semiconductors. The conductivity in intrinsic semiconductors is due to both the motion of electrons in the conduction band and the motion of holes in the valence band. The electrons in the valence band of intrinsic semiconductors need more energy to move to conduction band. This energy could be reduced by doping a pure semiconductor with certain impurities. The process of adding impurity is called doping. Such semiconductors are called extrinsic semiconductors. Based on the type of dopant extrinsic semiconductors are classified into n-type semiconductors and p-type semiconductors. The conductivity of extrinsic semiconductors is due to the motion of majority charge carries electrons in n-type and holes in p-type.

Doping of pentavalent impurity atoms to pure semiconductor results in the fifth electron which does not participate in bonding and loosely bound to the impurity atoms. The energy of these electrons lie in the energy gap of semiconductor.Thus donor level is created near the lower edge of the conduction band. Given a small amount of energy electrons move from donor level to conduction band. Thus there will be large number of free electrons in the conduction band. Thus electrons are the majority charge carriers in n-type semiconductors. Similarly doping of trivalent impurity atoms to pure semiconductor results in the formation of the acceptor level near the upper edge of the valence band. Holes are the majority carriers in p-type semiconductors.

9.2 Significance of Fermi level

Fermi level acts as a distinguishing energy position between filled and unfilled energy states in metals and semiconductors. In case of pure semiconductors, at normal temperatures, electrons are most probably found either in conduction band or in valence band. This is because of electrons in the top most levels of valence band absorb energy and move to conduction band. But electrons cannot spend more time there and hence they return to a lower energy level in valence band. Thus electrons will be under constant excitation and de-excitation process. Thus the average energy of the electron lies at the center of the energy gap and called Fermi energy.

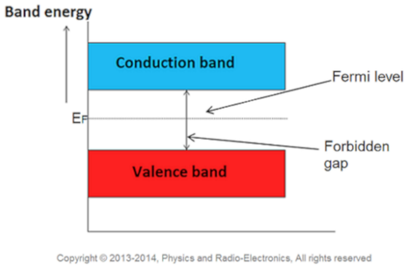


Figure 9.1: Fermi Level in Intrinsic Semiconductors

Figure 9.2: Fermi Level in Extrinsic semiconductors

In case of extrinsic semiconductors the Fermi energy lies towards conduction band in n-type and towards valence band in p-type semiconductors. Fermi level depends on temperature and changes with temperature.

9.3 Carrier Concentration in intrinsic semiconductors

Intrinsic semiconductor is the pure form of semiconductor. When sufficient energy is supplied to electrons in the valence band they absorb energy and move to conduction band leaving behind a hole. Thus the number of electrons in the conduction band is equal to number of holes in the valence band. The total number of electrons in the conduction band per unit volume of the semiconductor is known as electron concentration. Similarly number of holes in the valance band per unit volume of the semiconductor is known as hole concentration. The total number of charge carriers per unit volume in the semiconductor is called carrier concentration. In case of pure semiconductors electron concentration and hole concentration are same.

9.3.1 Expression for Electron concentration (N_e)

The following equation 9.1 determines the electron concentration in intrinsic semiconductors.

$$N_e = \frac{4\sqrt{2}}{h^3}(\pi m_e^* kT)^{\frac{3}{2}} e^{\left(\frac{E_F-E_g}{kT}\right)} \tag{9.1}$$

Here
h is Planck's constant
m_e^* is the effective mass of the electron
k is Boltzmann constant
T is absolute temperature
E_F is Fermi energy
E_g is the energy gap of the semiconductor.

9.3.2 Expression for Hole concentration (N_e)

The following equation 9.2 determines the electron concentration in intrinsic semiconductors.

$$N_h = \frac{4\sqrt{2}}{h^3}(\pi m_h^* kT)^{\frac{3}{2}} e^{\left(\frac{-E_F}{kT}\right)} \tag{9.2}$$

Here
h is Planck's constant
m_h^* is the effective mass of the electron
k is Boltzmann constant
T is absolute temperature
E_F is Fermi energy
E_g is the energy gap of the semiconductor.

9.3.3 Expression relating Fermi energy and Energy gap for an intrinsic semiconductor

The expressions for electron and hole concentrations in a pure semiconductor are given by equations 9.1 and 9.2

$$N_e = \frac{4\sqrt{2}}{h^3}(\pi m_e^* kT)^{\frac{3}{2}} e^{\left(\frac{E_F-E_g}{kT}\right)} \tag{9.3}$$

$$N_h = \frac{4\sqrt{2}}{h^3}(\pi m_h^* kT)^{\frac{3}{2}} e^{\left(\frac{-E_F}{kT}\right)} \tag{9.4}$$

Both electron concentration and hole concentration depend on fermi energy E_F. In case of intrinsic semiconductor $N_e = N_h$ and hence equating the equations 9.1 and 9.2 we get

$$(m_e^*)^{\frac{3}{2}} e^{\left(\frac{E_F-E_g}{kT}\right)} = (m_h^*)^{\frac{3}{2}} e^{\left(\frac{-E_F}{kT}\right)} \tag{9.5}$$

$$e^{\left(\frac{2E_F-E_g}{kT}\right)} = \left[\frac{m_h^*}{m_e^*}\right]^{\frac{3}{2}} \tag{9.6}$$

$$\frac{2E_F - E_g}{kT} = \frac{3}{2}\, ln\left(\frac{m_h^*}{m_e^*}\right) \tag{9.7}$$

$$2E_F - E_g = \frac{3}{2}\, kT\, ln\left(\frac{m_h^*}{m_e^*}\right) \tag{9.8}$$

$$E_F = \frac{E_g}{2} + \frac{3}{4}\, kT\, ln\left(\frac{m_h^*}{m_e^*}\right) \tag{9.9}$$

Thus the expression for Fermi energy in intrinsic semiconductors. If we consider that $m_h^* = m_e^*$ then equation 9.9 becomes

$$E_F = \frac{E_g}{2} \tag{9.10}$$

9.4 Expression for electrical conductivity in semiconductors

The conductivity in semiconductors is due two kinds of charge carries electrons and holes. Let us calculate the contribution of electrons for conduction.

Consider a semiconductor of area of cross section A and carrying current I. Let us assume that the current is only due to electrons. let v be the drift velocity of the electrons. Let N_e is the electron concentration. The current through the semi conductor is given by

$$I = \frac{q}{T}$$

Here q is the amount of charge crossing the given cross section in T second.

In unit time the distance l traveled by the electron will be numerically equal to drift velocity (v). Thus the number of electrons in the volume swept by the electrons in one second is the rate of flow of charge. The volume swept by the electrons in one second is given by Av. Since e is the charge on electron then, rate of flow of charge is given by

$$I = N_e e A v \tag{9.11}$$

We know that the expression for the current density is given by $j = \frac{I}{A}$. Thus the expression for current density is given by

$$j = N_e e v \tag{9.12}$$

The drift velocity v is related to the mobility of the electrons by the equation $v = \mu_e E$. Here μ_e is the mobility of electrons in semiconductor. E is the applied electric field strength. Thus substituting for v in equation 9.12 we get

$$j = N_e e \mu_e E \tag{9.13}$$

It is also known that

$$j = \sigma_e E \tag{9.14}$$

Here σ_e conductivity due to electrons. Comparing equations 9.13 and 9.14 we get

$$\sigma_e = N_e e \mu_e \tag{9.15}$$

Extending the same treatment for the conduction of holes we get an expression for electrical conductivity due to holes as

$$\sigma_h = N_h e \mu_h \tag{9.16}$$

Here μ_h is the mobility of holes.

Thus the electrical conductivity of the semiconductor σ is given

$$\sigma = \sigma_e + \sigma_h$$

$$\sigma = N_e e \mu_e + N_h e \mu_h$$

$$\sigma = e(N_e \mu_e + N_h \mu_h) \tag{9.17}$$

The equation 9.17 determines the electrical conductivity of semiconductors in general.

For an intrinsic semiconductors, we know that $N_e = N_h = n_i$. Thus the expressions for electrical conductivity of intrinsic semiconductor is given by

$$\sigma = n_i e(\mu_e + \mu_h) \tag{9.18}$$

Thus the expression for electrical conductivity in extrinsic and intrinsic semiconductors.

9.5 Hall Effect

Introduction This was observed first by E. H. Hall in the year 1879. When electric current is passed through a conductor or semiconductor placed in a magnetic field, a potential difference proportional to the current and to the magnetic field is developed across the material in a direction perpendicular to both the current and to the magnetic field. This effect is known as the Hall effect. The potential difference developed in a direction perpendicular to both current and magnetic field is called Hall voltage and the corresponding electric field is called Hall field.

Explanation Consider a conductor carrying current along +ve X axis. Magnetic field is applied along +ve Z axis. Thus electrons experience Lorentz force along the -ve Y axis. Thus electrons accumulate on the lower surface which results in the accumulation of +ve charges on the upper surface. This results in an electric field developed between the two surfaces. The force on the electrons due to electric field further apposes the force due to magnetic field. A stage is reached where in both the forces are equal and an equilibrium state is reached. The electric field developed across the material at equilibrium is called Hall Field.

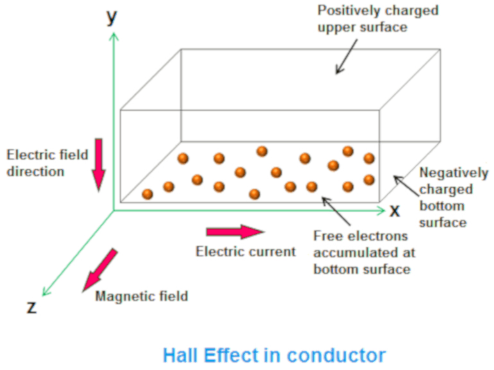


Figure 9.3: Hall effect in conductors

Expression for Hall Co-efficient Under equilibrium the Lorentz force on electrons is equal to force due to hall field

$$eE_H = Bev \tag{9.19}$$

Here e is electronic charge, E_H is Hall Field, B is Magnetic field, v is the velocity of electrons. The current density J is given by

$$J = nev \tag{9.20}$$

Here n is the numbmer density of charges. Dividing equation 9.19 by 9.20 we get

$$\frac{eE_H}{J} = \frac{Bev}{nev} \tag{9.21}$$

$$\frac{eE_H}{J} = \frac{B}{n} \tag{9.22}$$

$$E_H = \frac{BJ}{ne} \tag{9.23}$$

$$E_H = R_H BJ \tag{9.24}$$

Here R_H is called Hall coefficient and is given by

$$R_H = \frac{1}{ne} \tag{9.25}$$

The Hall voltage is given by the equation

$$V_H = E_H d = R_H BJd \tag{9.26}$$

Here d is the thickness of the material along y-axis.

Note: In case of n-type semiconductors R_H is negative and for p-type semiconductors R_H is positive.

Applications

1. This effect is the basis of many practical applications and devices such as magnetic field measurements, and position and motion detectors.

2. Hall effect is also used in the determination of type of extrinsic semiconductor (P-Type and N-Type).

3. Hall effect is also used to find semiconductor properties like carrier concentration and mobility of carriers.

Model Questions

1. Explain the significance of Fermi level in intrinsic and extrinsic semiconductors.

2. Write a note on carrier concentration in semiconductors.

3. Derive the relation between Fermi energy and energy gap for an intrinsic semiconductor.

4. Derive and expression for electrical conductivity in intrinsic and semiconductors.

5. Describe hall effect. Derive an expression for hall co-efficient.

Numerical Problems

1. The electron and hole mobilities of silicon are $0.14 m^2 V^{-1} s^{-1}$ and $0.05 m^2 V^{-1} s^{-1}$ respectively at a certain temperature. If the electron concentration in silicon is $1.5 \times 10^{16} electrons/m^3$ calculate the resistivity of silicon. Ans. $2193 \Omega m$.

2. The resistivity of silicon at $27C$ is $3000 \Omega m$. Assuming electron and hole mobilities of 0.17 and 0.35 $m^2 V^{-1} s^{-1}$ respectively, calculate the intrinsic carrier concentration at $27C$. Ans. $1.016 \times 10^{16} electrons/m^3$

3. The mobilities of holes and electrons in an extrinsic germanium at 300K is given by 0.2 $m^2 V^{-1} s^{-1}$ and 0.36 $m^2 V^{-1} s^{-1}$. If the electron concentration and hole concentrations are $1.8 \times 10^{20} electrons/cm^3$ and $1.6 \times 10^5 electrons/cm^3$ estimate the resistivity of germanium at room temperature.Is it p-type or n-type? Ans. $9.645 \times 10^{-5} \Omega m$, n-type.

4. Calculate the concentrations at which the acceptor atoms must be added to a germanium sample to get a p-type semiconductor with conductivity 0.15 per ohm-m. Given the mobility of holes $0.17 m^2 V^{-1} s^{-1}$. Ans. $N_h = 5.5 \times 10^{18} m^{-3}$

5. The conductivity and Hall co-efficient of an n-type silicon specimen are 112/ohm-m and $1.25 \times 10^{-3} m^3/C$, respectively. Calculate the charge carrier concentration and electron mobility.Ans. $N_e = 5 \times 10^{21} m^{-3}$, $\mu_e = 0.14$.

6. The hall co-efficient of a material is $-3.68 \times 10^{-5} m^3/C$. What is the type of charge carriers? Also calculate the carrier concentration.Ans. R_H is - Ve, electrons, $1.7 \times 10^{23}/m^3$

7. The hall co-efficient of a specimen of a doped silicon is found to be $3.66 \times 10^{-4}/coulomb$. The resistivity of the specimen is $9.93 \times 10^{-3} ohm - m$. Find the mobility and density of charge carrier, Assuming single carrier concentration. Ans. $1.7055 \times 10^{22}/m^3$ and $0.041 m^2 V^{-1} s^{-1}$

8. A rectangular plane sheet of semiconductor material has dimensions 2cm along Y-direction and 1mm along Z-direction. Hall probes are attached on its surfaces parallel to X-Y plane and a magnetic field of flux density $1 weber/m^2$ is applied along Z-direction. A current of 3 mA is flowing in it in the X-direction. Calculate the hall voltage measured by the probe, if the hall co-efficient of the material is $3.66 \times 10^{-4}/coulomb$. Also calculate the charge carrier concentration. Ans. $E_H = 0.0549 Vm^{-1}, V_H = 1.1 mV, n = 1.71 \times 10^{22}/m^3$.